Phase Transitions and Collective Phenomena

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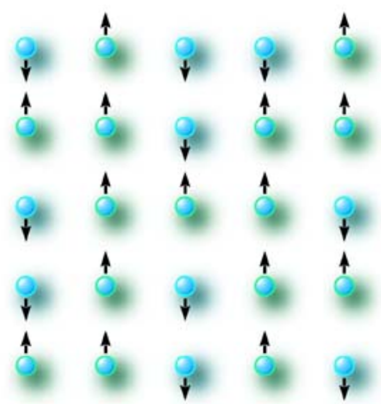
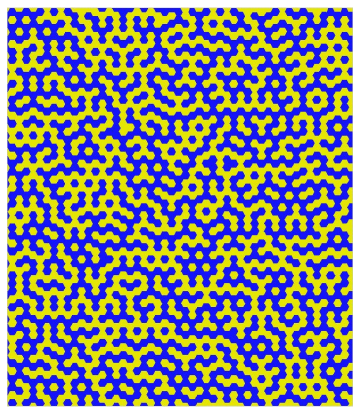
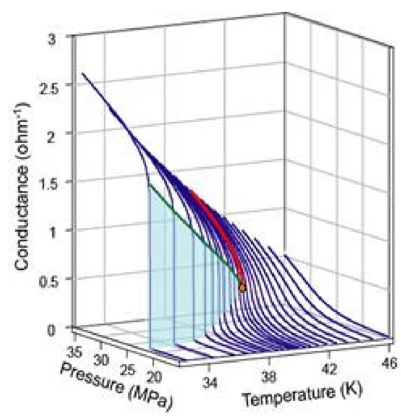
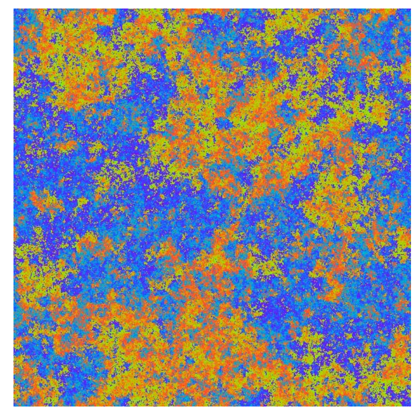
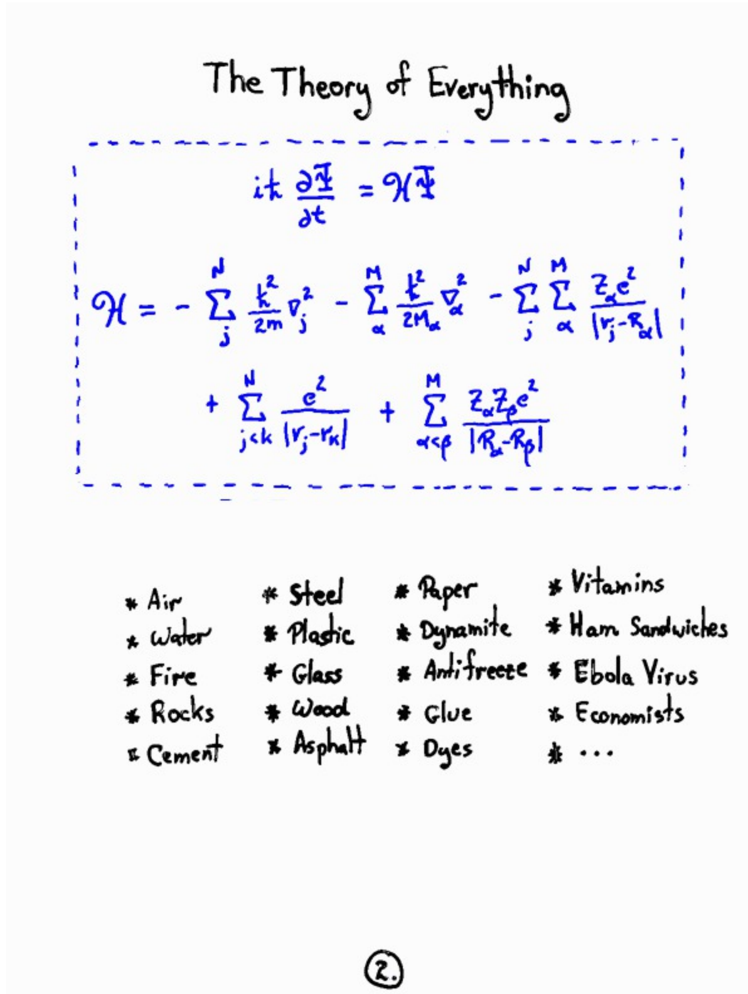






Pyrrhic victories of Theories of Everything



How can we describe complex physical systems?

▷ e.g. molecules in a liquid? electrons in solid? spins on a lattice?

Microscopic

↓

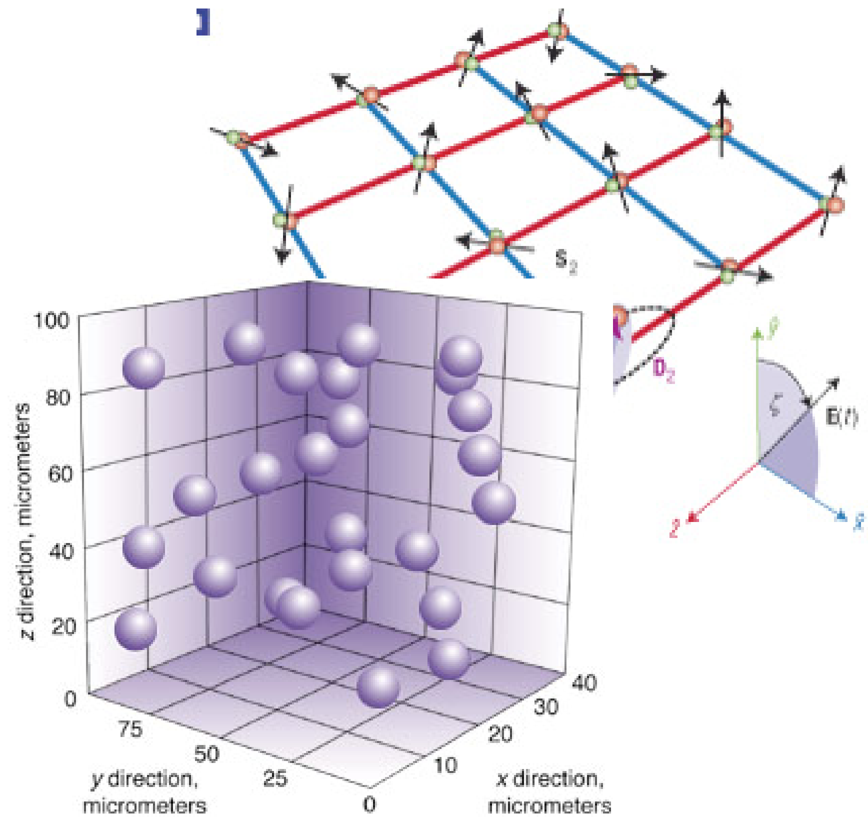
Microstates

$\{p_i, q_i\}$, $\{\sigma_i\}$

↓

Microscopic Hamiltonian

(Classical/Quantum)



How can we describe complex physical systems?

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Microscopic	$\longleftrightarrow$	Macroscopic
$\downarrow$		$\downarrow$
Microstates		Macrostate fns.
$\{p_i, q_i\}$, $\{\sigma_i\}$		P, V, T, S, E
$\downarrow$		$\downarrow$
Microscopic Hamiltonian (Classical/Quantum)		Laws of Thermodynamics

How can we describe complex physical systems?

▷ e.g. molecules in a liquid? electrons in solid? spins on a lattice?

MICROSCOPIC	$\longleftrightarrow$	MACROSCOPIC
$\downarrow$		$\downarrow$
Microstates		Macrostate fns.
$\{p_i, q_i\}$, $\{\sigma_i\}$		P, V, T, S, E
$\downarrow$		$\downarrow$
Microscopic Hamiltonian (Classical/Quantum)		Laws of Thermodynamics

Connection provided by Statistical Mechanics:

▷ Partition function $\mathcal{Z} = \sum_{\{\mu\}} e^{-\beta H[\{\mu\}]}$, $\quad P(\{\mu\}) = \frac{e^{-H[\{\mu\}]/k_B T}}{\mathcal{Z}}$

"More is Different"

▷ Non-interacting system: $\mathcal{Z} = \mathcal{Z}_1 \mathcal{Z}_2 \cdots \mathcal{Z}_{N\to\infty}, \to$ Ideal Gas Laws

▷ Interactions: singularities in $\mathcal{Z}$

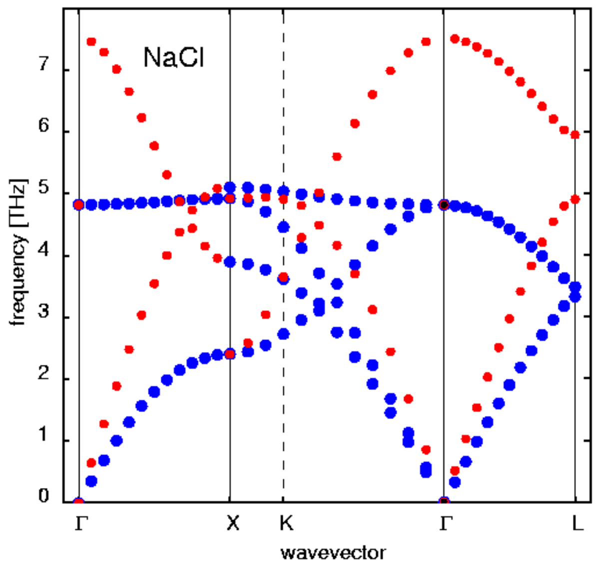
$\to$ Phase transitions to new phases of matter...

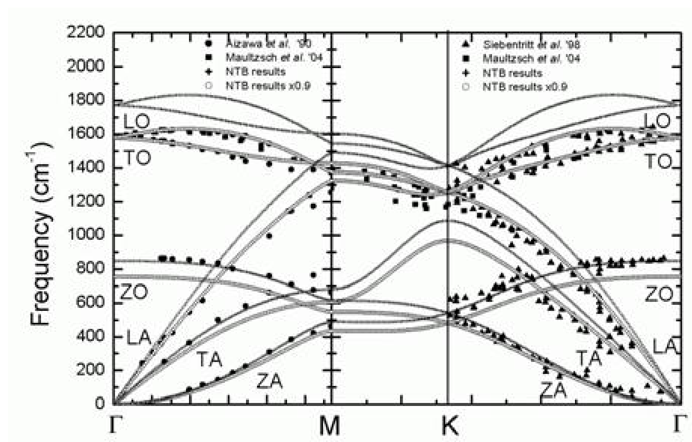
classical media	solid-liquid-gas, etc.
'soft' matter	liquid crystals, etc.
'quantum' matter	superconductivity, magnetism, etc.
high energy astrophysics	baryogenesis, etc.

Transitions signalled by symmetry breaking

$\to$ low-energy collective excitations (e.g. phonons, spin-waves, etc.) and Universality

Broken Symmetry





▷ Phonons are independent of the type of bonds!

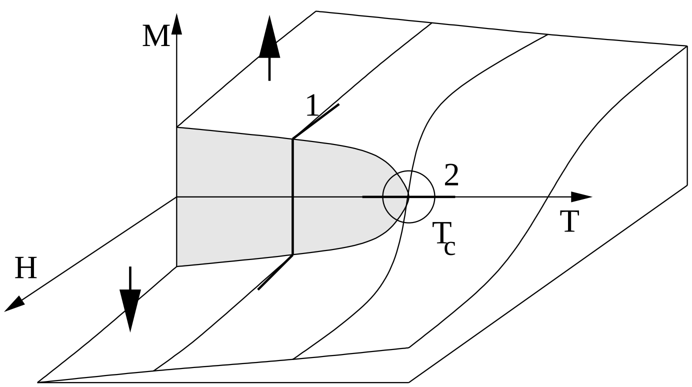
▷ Effective theories are not sensitive to details of the microscopic laws.

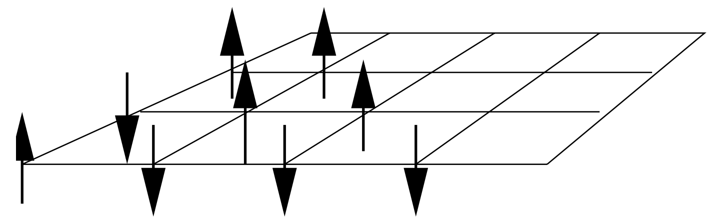
▷ Conversely, inference of high energy atomic bonding and microscopics are impossible using low energy sound waves.

Phase Transitions

▷ Two important classes, cf. phase diagram of classical Ising Ferromagnet

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j + H_{\text{ext}} M, \qquad M = \sum_i \sigma_i$$





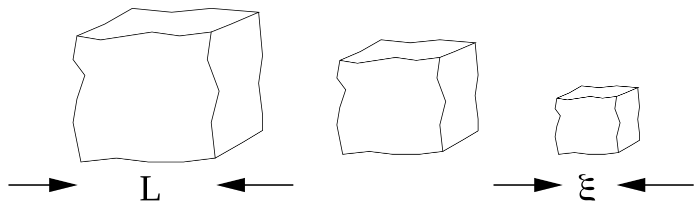
1. First order: Discontinuous change of 'order parameter' M

2. Second order: order parameter grows continuously

Nature of Critical Point?

▷ In quantum/classical statistical mechanics
continuous phase transitions play very special role... why?

Consider the correlation length ξ?...



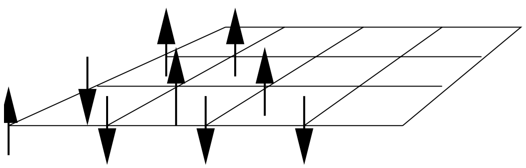
...length scale over which fluctuations are correlated.

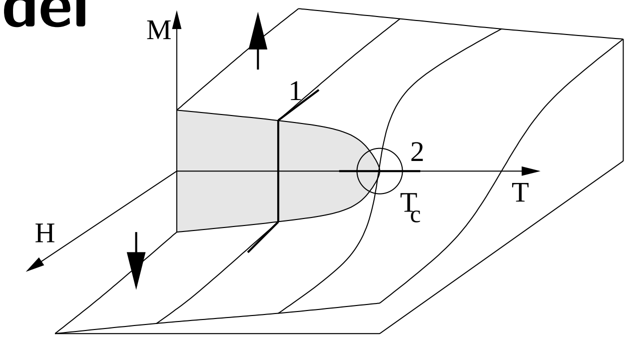
▷ At critical point T_c, correlation length ξ diverges...

$$\xi \sim |t|^{-\nu}, \qquad t = \frac{T - T_c}{T_c}$$

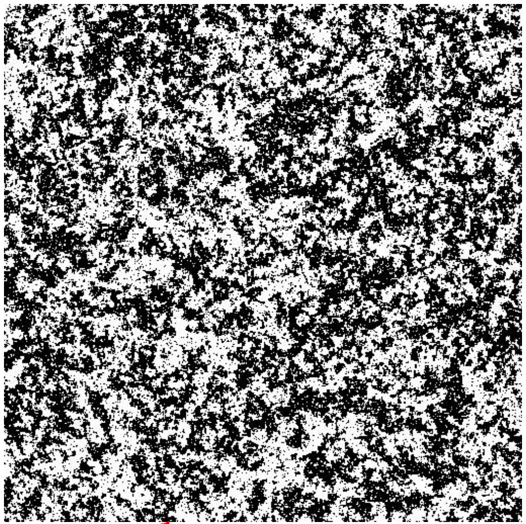
...and thermodynamic properties become singular.

Example: Ising model

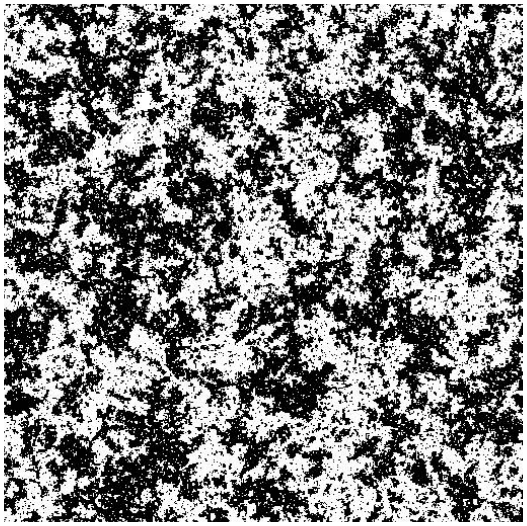




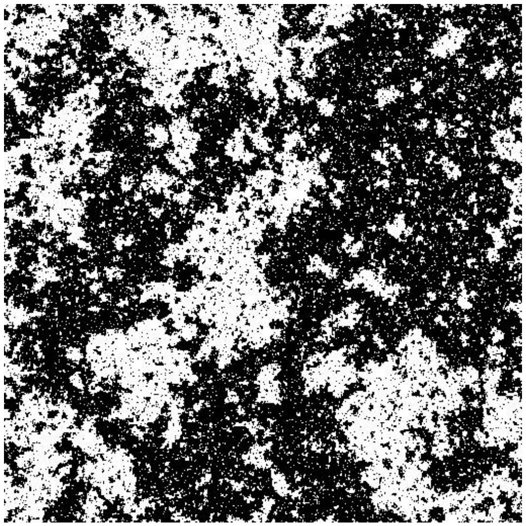
$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$



$\beta = \frac{1}{k_B T} = 0.2$



$\beta = 0.21$

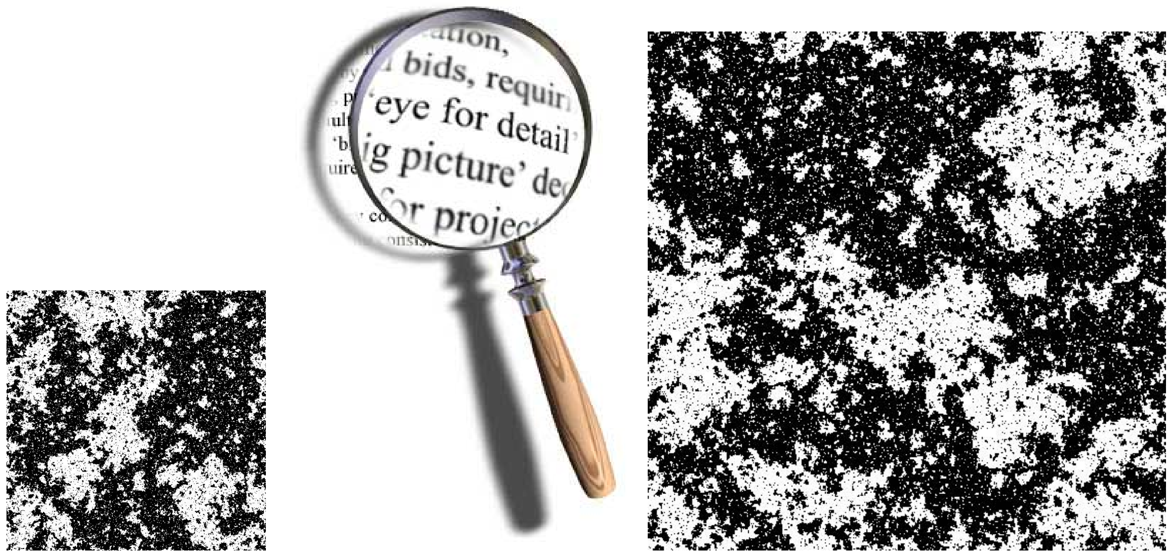


$\beta = 0.219$

Consequences

▷ Universality: since $\xi \to \infty$, microscopic scales become redundant at T_c!

▷ Scaling and self-similarity: Since there is no characteristic length scale at the critical point, correlation functions become scale invariant.



motivates "coarse-grained" theory based on **only** fundamental symmetries (rotation, translation, etc.) — Ginzburg-Landau phenomenology

Aim of course

to motivate, develop, & analyse critical phenomena
in framework of Ginzburg-Landau phenomenology

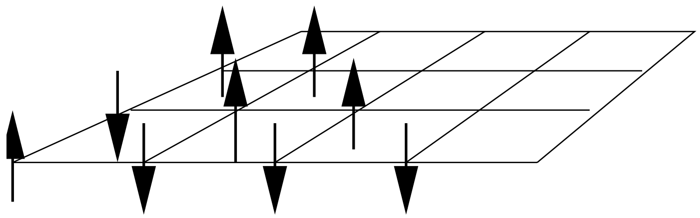
Approaches: mean-field theory
broken symmetry & fluctuations
scaling theory
field theory & renormalisation group

N.B. with connection to QFT

Example: Classical Ising Ferromagnet

▷ Microscopic Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$



▷ Coarse-grained order parameter :

local magnetisation $m(\mathbf{r}_i) = \frac{1}{R^d} \sum_{|\mathbf{r}_j - \mathbf{r}_i| < R} \sigma_j$

▷ Ginzburg-Landau phenomenology:

$$\beta H = \int d^d r \left[\frac{t}{2} m^2 + \frac{K}{2} (\nabla m)^2 + u m^4 + \cdots \right], \qquad \mathcal{Z} = \int Dm(\mathbf{r}) e^{-\beta H}$$

...compatible with symmetries

Phenomenology

▷ Phase transition (in dimensions $d > 1$) to ferromagnet at $t = 0$

▷ Spontaneous symmetry breaking:

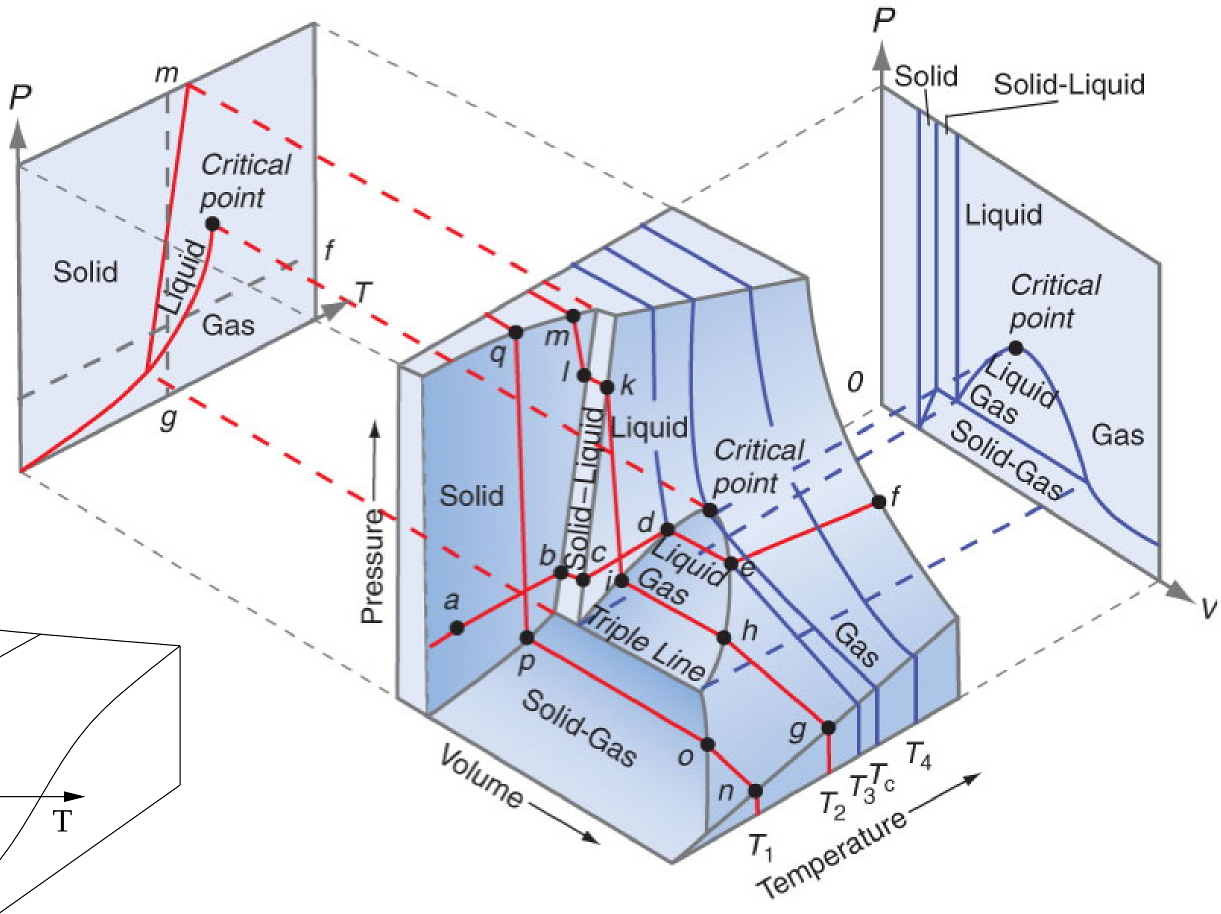
$$\begin{cases} \langle m \rangle_T = 0 & t > 0 \\ \langle m \rangle_T \sim |t|^\beta & t < 0 \end{cases}, \qquad \text{i.e.} \qquad t = \frac{T - T_c}{T_c}$$

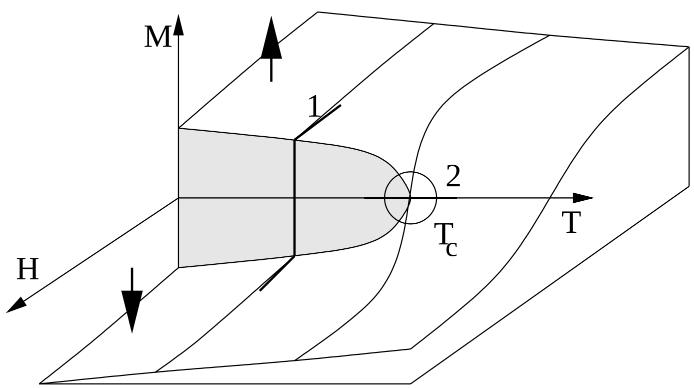
▷ Critical phenomena: $\xi \sim |t|^{-\nu}$, $\frac{\partial m}{\partial h} \sim |t|^{-\gamma}$, etc.;
at T_c, $\langle m(\mathbf{r}) m(0) \rangle_T \sim |\mathbf{r}|^{-(d-2+\eta)}$

▷ Ising Universality class:

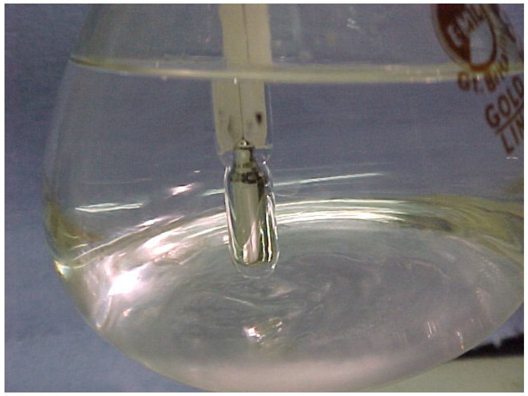
includes uniaxial ferro- and antiferromagnet, liquid-gas,
Mott-Hubbard metal-insulator transition, etc.

▷ Liquid-gas transition

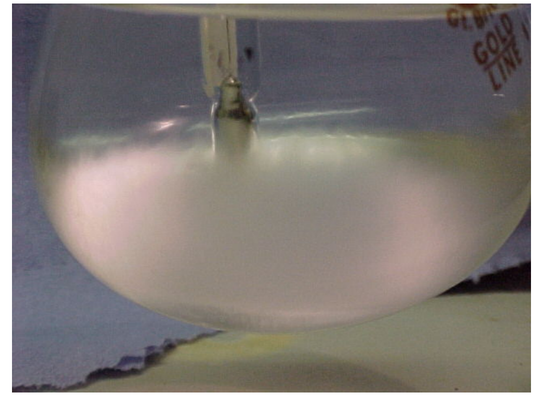




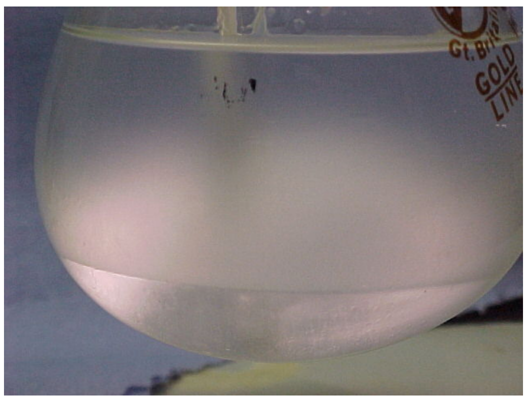
Critical Opalescence



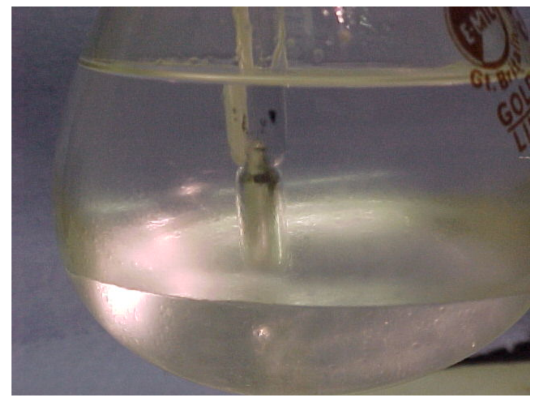
$T > T_c$



$T \simeq T_c$



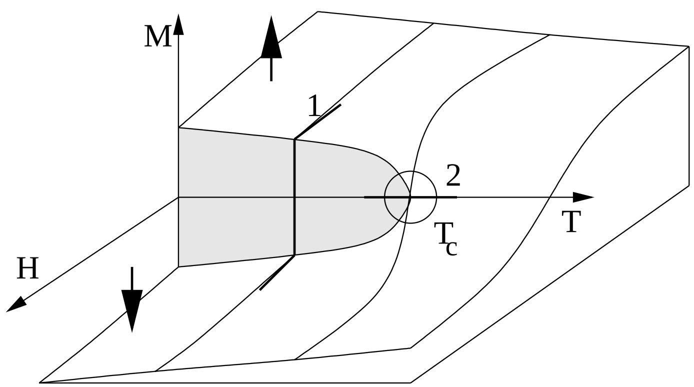
$T < T_c$

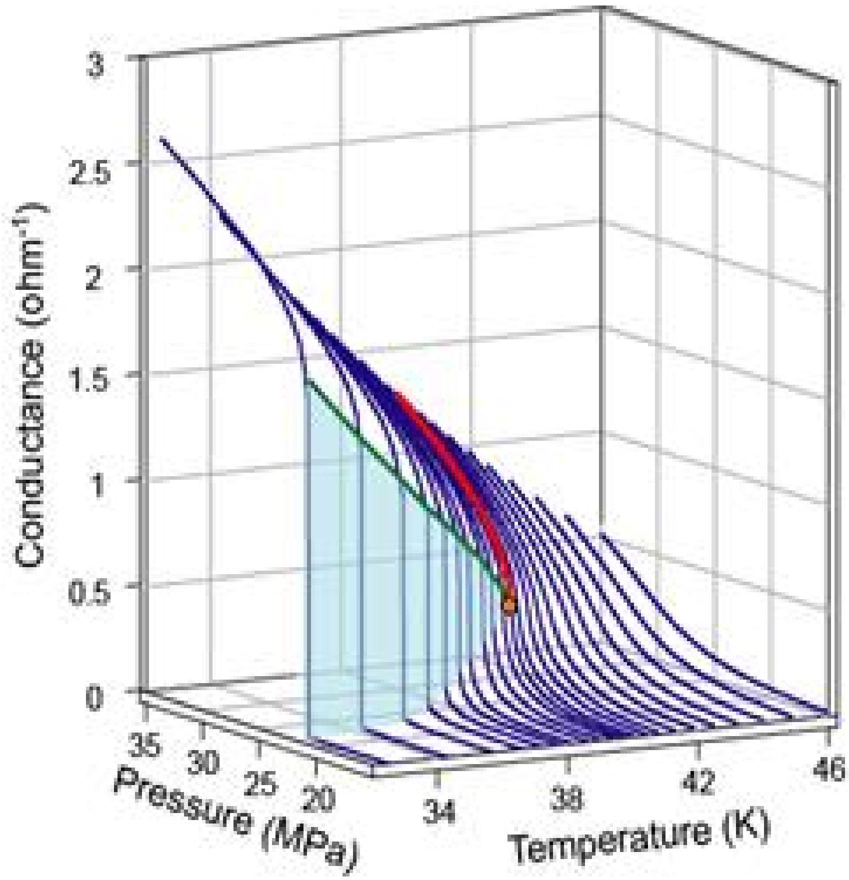


$T \ll T_c$

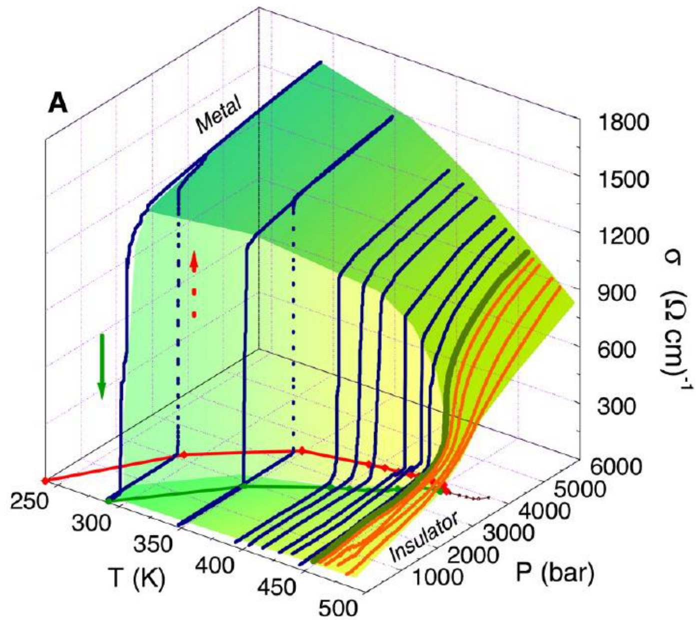
- Hexane and Methanol mixture through critical point

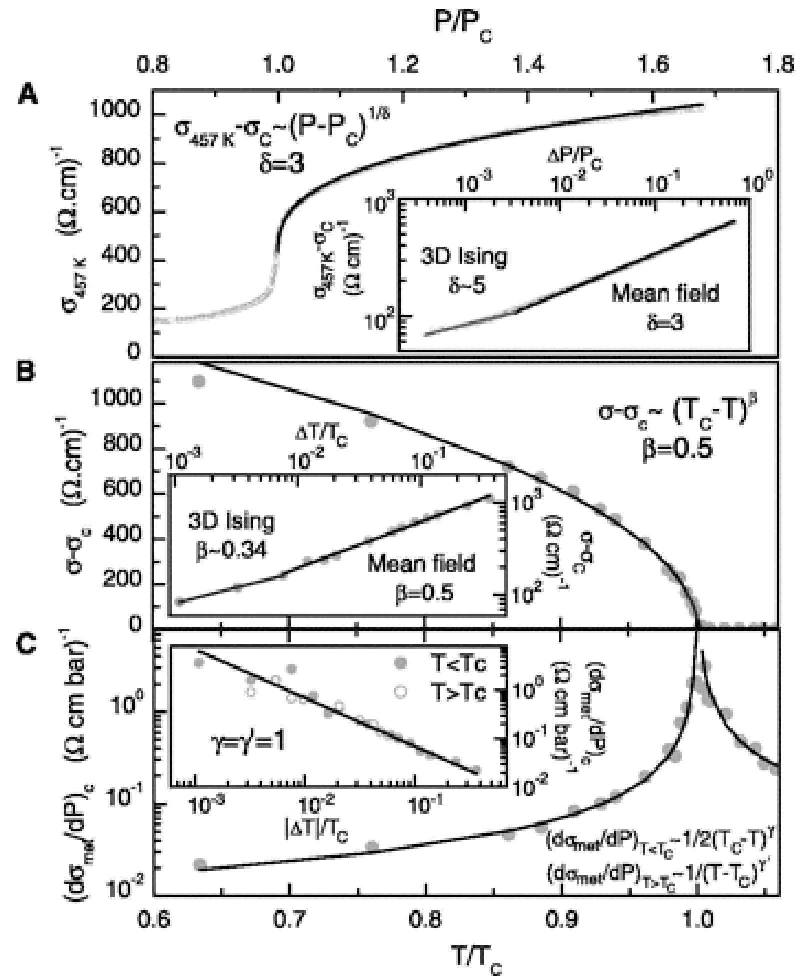
▷ Mott-Hubbard metal-insulator transition





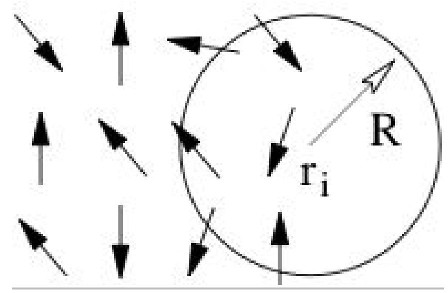
Critical exponents





▷ V_2O_3 Metal-Insulator transition

Example: Classical XY-Ferromagnet



▷ Microscopic Hamiltonian

$$H = -J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j, \qquad \mathbf{S}_i = (\cos\theta_i, \sin\theta_i)$$

▷ Order parameter: local magnetisation $\mathbf{m}(\mathbf{r}_i) = \dfrac{1}{R^d} \sum_{|\mathbf{r}_j - \mathbf{r}_i| < R} \mathbf{S}_j$

▷ Ginzburg-Landau phenomenology

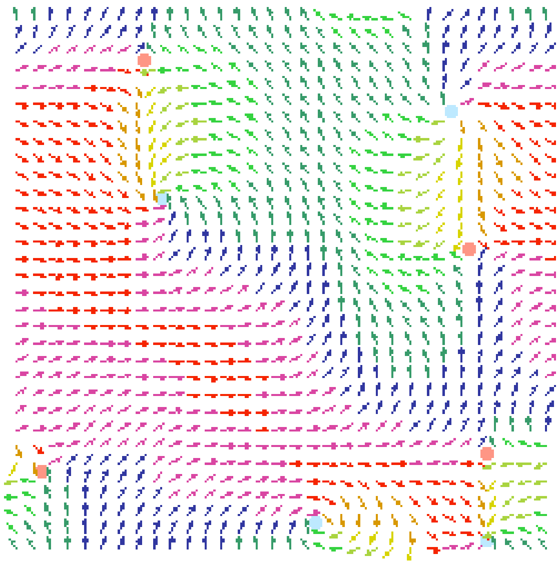
$$\beta H = \int d^d r \left[\frac{t}{2}\mathbf{m}^2 + \frac{K}{2}(\nabla \mathbf{m})^2 + u(\mathbf{m}\cdot\mathbf{m})^2 + \cdots \right]$$

Phenomenology

▷ Phase transition (in dimensions $d > 2$) to ferromagnet at $t = 0$

▷ Spontaneous symmetry breaking:

$$\begin{cases} \langle \mathbf{m} \rangle_T = 0 & t > 0 \\ \langle \mathbf{m} \rangle_T \sim |t|^\beta & t < 0 \end{cases}, \qquad \xi \sim |t|^{-\nu}$$



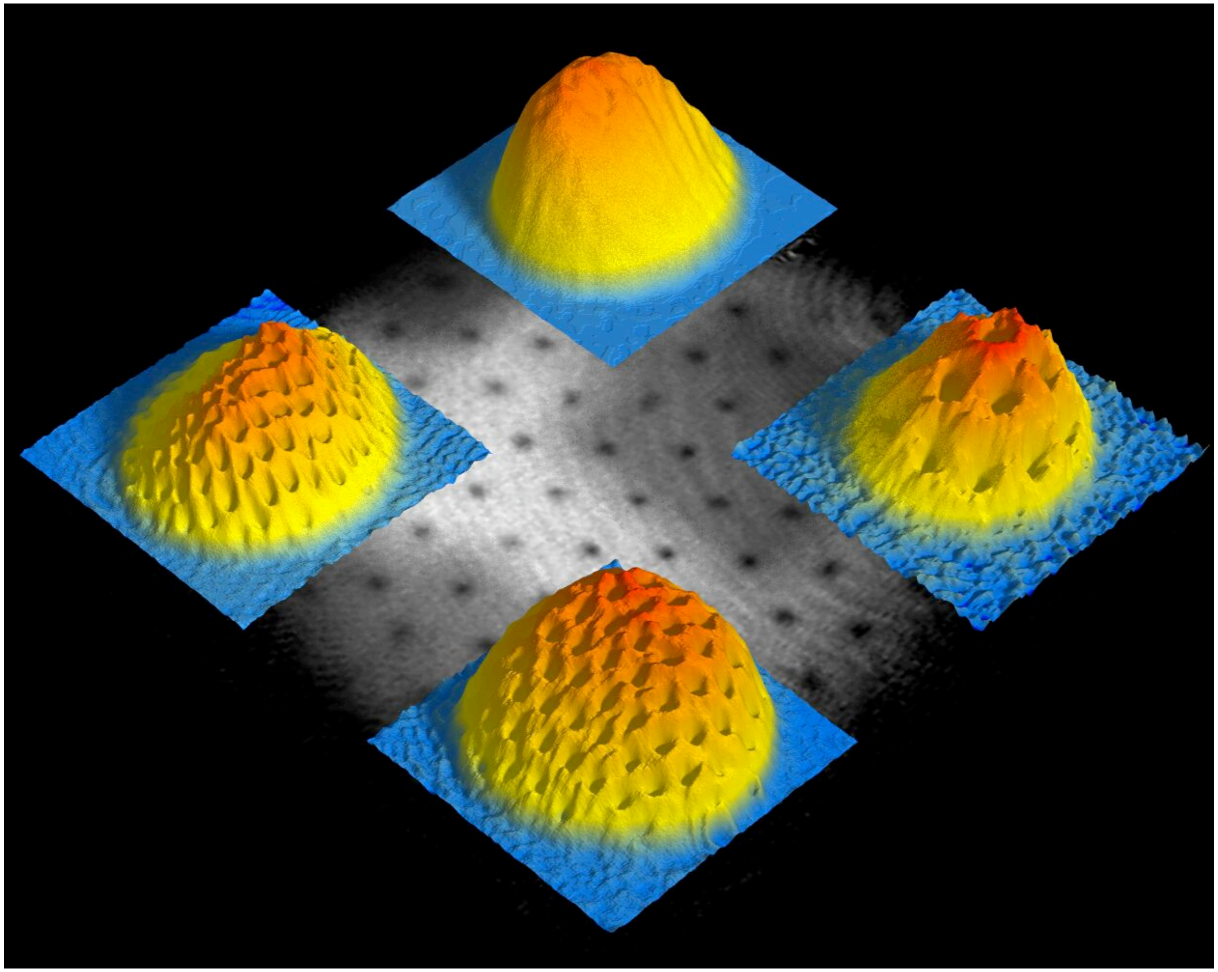
▷ Low-energy collective fluctuations — spin-waves

▷ Vortex configurations
→ topological phase transition in $d = 2$

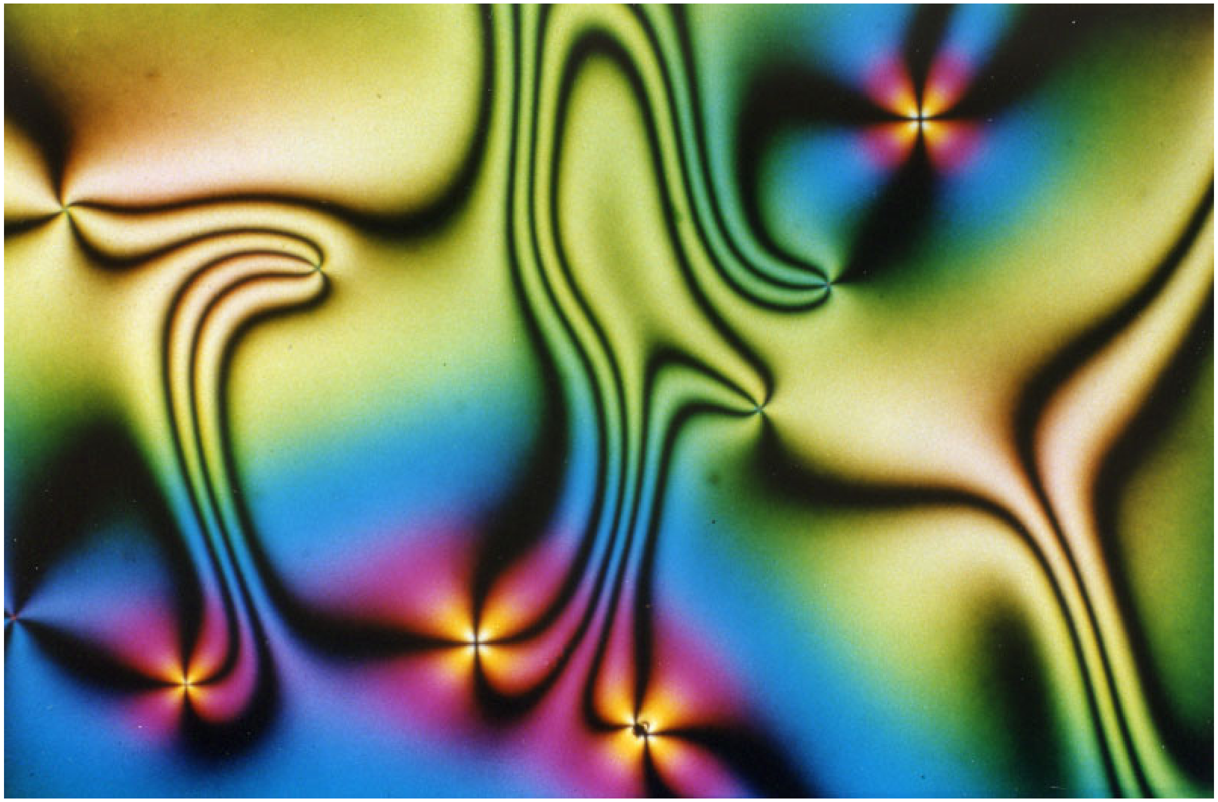
▷ XY Universality Class

includes superconductors and superfluids,
melting in two-dimensions, etc.

e.g. Vortices in atomic superfluid



e.g. disclinations in a liquid crystal



Phase Transitions and Collective Phenomena

- ▷ Synopsis
- ▷ What's missing?

  Non-equilibrium, experimental justification, applications outside condensed matter (e.g. HEP, biology,. . .)
- ▷ Prerequistes?

  Statistical Mechanics; Functional methods (useful, but not assumed)
- ▷ Lecture notes (`www`), problem sets and supervisions
- ▷ Books

Synopsis

- ▷ INTRODUCTION TO CRITICAL PHENOMENA: Concept of Phase Transitions; Order Parameters; Response Functions; Universality. [1]
- ▷ GINZBURG-LANDAU THEORY: Mean-Field Theory; Critical Exponents; Symmetry Breaking, Goldstone Modes, and the Lower Critical Dimension; Fluctuations and the Upper Critical Dimension; Importance of Correlation Functions; Ginzburg Criterion. [3]
- ▷ SCALING: Self-Similarity; The Scaling Hypothesis; Kadanoff's Heuristic Renormalisation Group (RG); Gaussian Model; Fixed Points and Critical Exponent Identities; Wilson's Momentum Space RG; Relevant, Irrelevant and Marginal Parameters; †ϵ-expansions. [4]
- ▷ TOPOLOGICAL PHASE TRANSITIONS: Continuous Spins and the Non-linear σ-model; XY-model; Algebraic Order; Topological defects, Confinement, the Kosterlitz-Thouless Transition and †Superfluidity in Thin Films. [2]
- ▷ QUANTUM PHASE TRANSITIONS: Classical/Quantum Mapping; the Dynamical Exponent; Quantum Rotors; †Haldane Gap; †Asymptotic Freedom; †Quantum Criticality. [2]

Phase Transitions and Collective Phenomena

- Synopsis
- Prerequistes?

  Statistical Mechanics; Functional methods (useful, but not assumed)
- Lectures — Moodle
- Lecture notes (`www`) & problem sets
- Supervisions
- Books

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Books

- J. Cardy, **Scaling and Renormalisation in Statistical Physics**, (Cambridge University Press, Lecture Notes in Physics), 1996.
- *P. M. Chaikin and T. C. Lubensky, **Principles of Condensed Matter Physics** (CUP) 1995.
- *M. Kardar, **Statistical Physics of Fields**, (CUP) 2007.
- L. P. Kadanoff, **Theories of Matter: Infinities and Renormalization**, arXiv:1002.2985v1 [physics.hist-ph], 2010.
- J. Zinn-Justin, **Phase Transitions and Renormalization Group**, (Oxford Graduate Texts) 2013.