Phase Transitions and Collective Phenomena

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Preface

The fundamental goal of **statistical mechanics** is to provide a framework in which the **microscopic** probabilistic description of systems with large numbers of degrees of freedom (such as the particles which constitute a gas) can be reconciled with the description at the **macroscopic** level (using equilibrium state variables such as pressure, volume and temperature). When we first meet these ideas they are usually developed in parallel with simple examples involving collections of weakly or non-interacting particles. However, strong interactions frequently induce transitions and lead to new equilibrium phases of matter. These phases exhibit their own characteristic fluctuations or elementary excitations known as **collective modes**. Although a description of these phenomena at the microscopic level can be quite complicated, the important large scale, or long-time "hydrodynamic" behaviour is often simple to describe. Phenomenological approaches based on this concept have led to certain **quantum** as well as **classical field theories** that over recent years have played a major role in shaping our understanding of condensed matter and high energy physics.

The goal of this course is to motivate this type of description; to establish and begin to develop a framework in which to describe critical properties associated with classical and quantum phase transitions; and, at the same time, to emphasise the importance and role played by **symmetry** and **topology**. Inevitably there is insufficient time to study such a wide field in any great depth. Instead, the aim will always be to develop fundamental concepts.

The phenomenological **Ginzburg-Landau theory** has played a pivotal rôle in the development of our understanding *critical phenomena* in both classical and quantum statistical mechanics, and much of our discussion will be based on it. The majority of the course will be involved in developing the important concept of **universality** in statistical mechanics and establish a general framework to describe critical phenomena — the **scaling theory** and the **renormalisation group**.

Synopsis

- ▷ INTRODUCTION TO CRITICAL PHENOMENA: Concept of Phase Transitions; Order Parameters; Response Functions; Universality. [1]
- ▷ GINZBURG-LANDAU THEORY: Mean-Field Theory; Critical Exponents; Symmetry Breaking, Goldstone Modes, and the Lower Critical Dimension; Fluctuations and the Upper Critical Dimension; Importance of Correlation Functions; Ginzburg Criterion. [3]
- ▷ SCALING: Self-Similarity; The Scaling Hypothesis; Kadanoff's Heuristic Renormalisation Group (RG); Gaussian Model; Fixed Points and Critical Exponent Identities; Wilson's Momentum Space RG; Relevant, Irrelevant and Marginal Parameters; $^\dagger\epsilon$-expansions. [4]
- ▷ TOPOLOGICAL PHASE TRANSITIONS: Continuous Spins and the Non-linear σ-model; XY-model; Algebraic Order; Topological defects, Confinement, the Kosterlitz-Thouless Transition and †Superfluidity in Thin Films. [2]
- ▷ QUANTUM PHASE TRANSITIONS: Classical/Quantum Mapping; the Dynamical Exponent; Quantum Rotors; †Haldane Gap; †Asymptotic Freedom; †Quantum Criticality. [2]

Material indicated by a † will be included if time allows.

Bibliography

[1] *M. Kardar, **Statistical Physics of Fields**, Cambridge University Press, 2007. *Readers will note the non-coincidental similarity of presentation of these notes and lectures with this book.*

[2] J. Cardy, **Scaling and Renormalisation in Statistical Physics**, (Cambridge University Press, Lecture Notes in Physics), 1996. *An extremely clearly written and concise exposition of the fundamental concepts in statistical physics.*

[3] *P. M. Chaikin and T. C. Lubensky, **Principles of Condensed Matter Physics** (CUP) 1995. *An accessible book including applications of modern statistical field theory mostly to problems in "soft condensed matter physics".*

[4] R. P. Feynman, **Statistical Mechanics**, Benjamin, New York, (1972). *Another classic text by Feynman which covers the basic concepts.*

[5] N. Goldenfeld, **Lectures on Phase Transitions and the Renormalization Group**, Westview Press 1992. *A readable series of lectures at the right level for this course.*

[6] C. Itzykson and J.-M. Drouffle, **Statistical Field Theory, Vols. 1 & 2**, (CUP) 1991. *Rather advanced to be directly useful for this course.*

[7] N. D. Mermin, Rev. Mod. Phys. **51**, 591 (1979). *Excellent introduction to the concept of phase transitions driven by topological defects.*

[8] G. Parisi, **Statistical Field Theory**, (Frontiers in Physics, Lecture Note Series) 1988.

[9] L. P. Kadanoff, **Theories of Matter: Infinities and Renormalization**, arXiv:1002.2985v1 [physics.hist-ph], 2010. *A good historical and philosophical introduction to the subject matter from one of the players in the field.*

[10] J. Zinn-Justin, **Phase Transitions and Renormalization Group**, (Oxford Graduate Texts) 2013. *Exceptionally complete mathematical and physical treatment, but perhaps less introductory than the above texts.*

This course follows no particular text but a number of books may be useful. Those which are particularly useful are marked by a "*" in the list. I wish to thank Prof. Simons and Dr Kwasigroch for earlier versions of these notes.

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