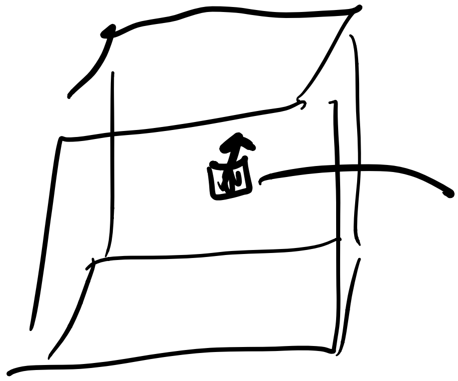
Define coarse-grained magnetisation field



$\underline{m}(\underline{x})$

"i.e. we have integrated out the short-wavelength scale fluctuations of local magnetisation"

- works if $\underline{m}(\underline{x})$ is sufficiently slowly varying

$$\underline{x} = (x_1, \ldots, x_d)^T$$

n-component spins $\underline{m} = (m_1, \ldots, m_n)^T$

$n=3$ isotropic : Heisenberg

$n=2$ planar : XY model, superconductivity
superfluidity

$n=1$ uniaxial : Ising
liq $\leftrightarrow$ gas

$$Z = \mathrm{tr}\, e^{-\beta H_{mic}} = \int \mathcal{D}m\, e^{-\beta H[m]}$$

too difficult

Conditions on construction of effective field theory

1.). Locality

$$\beta H = \int d^d\underline{x}\; f[\underline{m}(\underline{x}), \nabla \underline{m}, \ldots]$$

i.e. interactions are sufficiently short-ranged.

2. Global rotational symmetry in spin space (for $h=0$)

$$\beta H[\underline{m}] = \beta H[R_m \underline{m}(\underline{x})]$$

3. Translational and rotation invariance in real space.

$\underline{m}\cdot\underline{m}$ $\quad$ $(\underline{m}\cdot\underline{m})^2$

$$\rightsquigarrow \beta H[\underline{m}(\underline{x})] = \int d^d\underline{x}\left[\frac{t}{2} m^2 + u m^4, \ldots\right.$$

$$+ \frac{K}{2}(\nabla \underline{m})^2 + \frac{L}{2}(\nabla^2 \underline{m})^2 + \cdots$$

$$- \underline{h} \cdot \underline{m} \,]$$

Ginzburg-Landau Hamiltonian

$$\frac{\partial m_i}{\partial x_j} \frac{\partial m_i}{\partial x_j}$$

could in principle have an infinite number of terms

- near T_c, expansion in small m
  $\rightsquigarrow$ higher order terms are truncated.
- "let the theory be simplest but not any simpler."

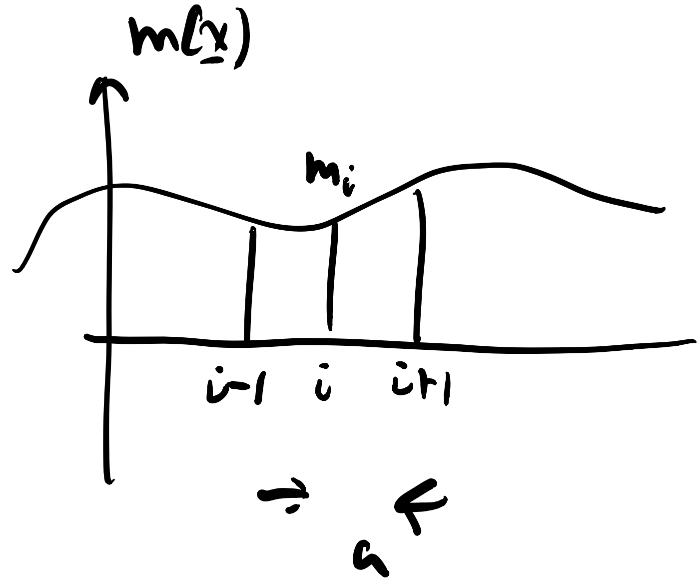
Partition function

$$Z(T, h) = \int \mathcal{D} m(\underline{x}) \, e^{-\beta H[\underline{m}]}$$

functional integral.

$$\int \mathcal{D}m\; f(\underline{m}, \nabla \underline{m}, \dots)$$

$$\equiv \lim_{\substack{a \to 0 \\ N^d \to \infty}} \int \prod_{i=1}^{N^d} dm_i\; f\left(m_i, \frac{m_{i+1}-m_i}{a}, \dots\right)$$



Note t, u, K etc are all functions of original microscopic parameters and T

+ external parameters (e.g. pressure.

Landau mean field theory

$$Z(h,T) \equiv e^{-\beta F[h,T]} = \int \mathcal{D}m\; e^{-\beta H[m]}$$

~ max of

integral

Saddle - pt. approximation

$$\beta F = \min_{\underline{m}} \left[\beta H[\underline{m}] \right]$$

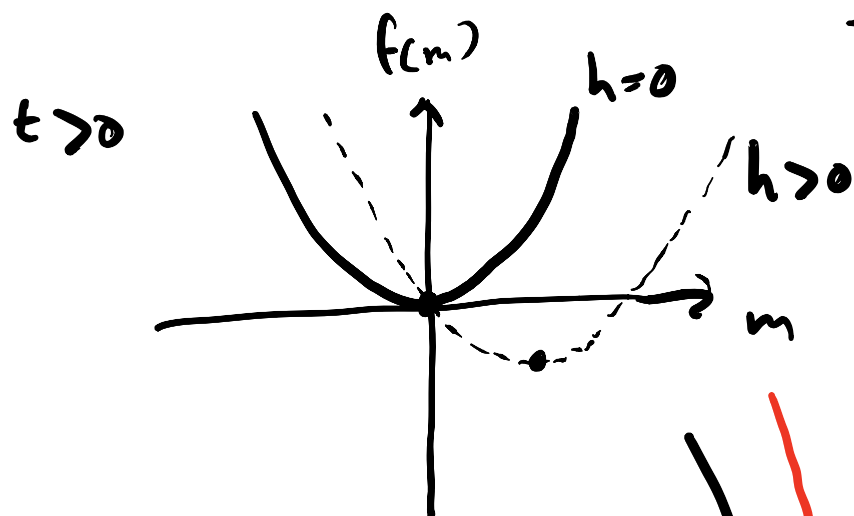
For $K>0$, minimum occurs when "KE" is constant

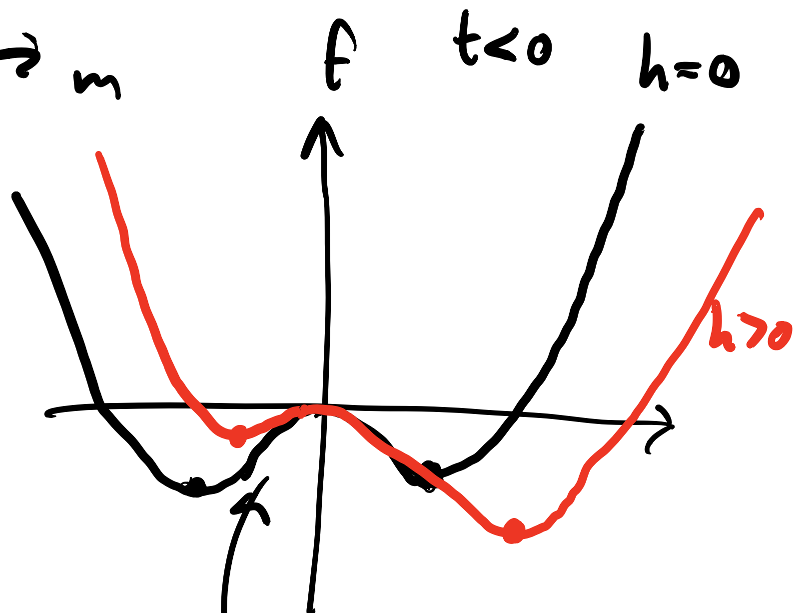
$$\underline{\nabla}\, \underline{m} \equiv 0$$

$$\underline{m}(\underline{x}) = \bar{\underline{m}} = \bar{m}\, \hat{\underline{e}}_h$$

$$\frac{\beta F}{V} = \min_{\underline{m}} \left[\frac{t}{2} m^2 + u m^4 - \underline{h} \cdot \underline{m} \right]$$

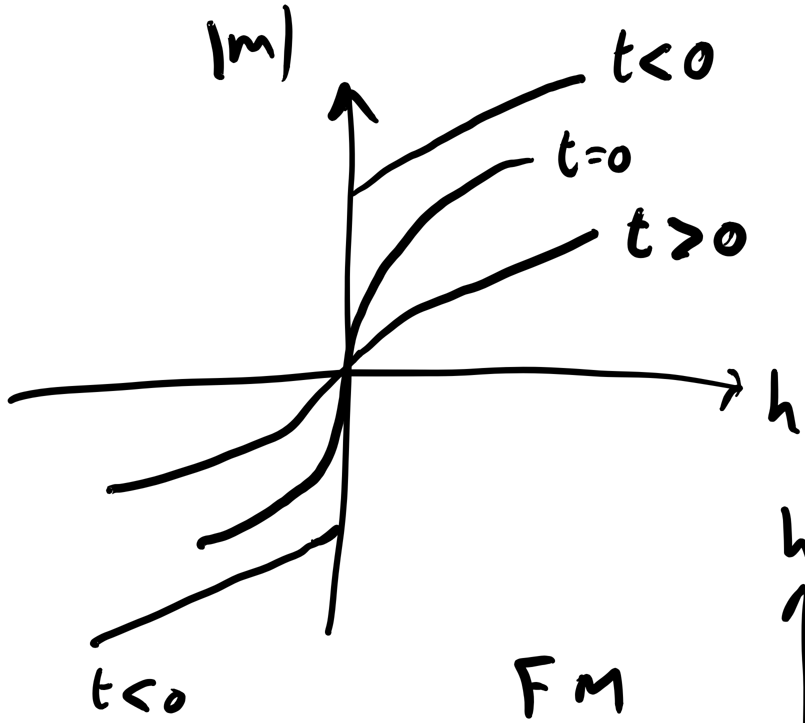
$f(m)$

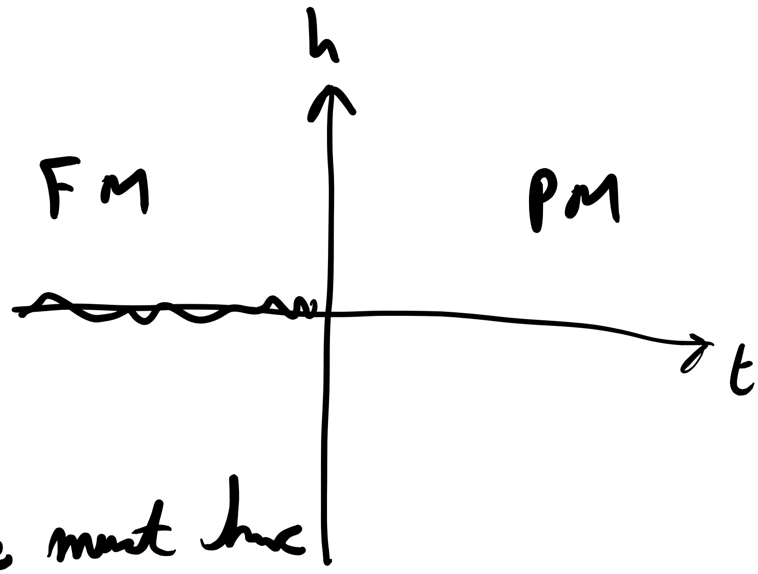




Spontaneous symmetry break

$u>0$ for stability





Phenomenologically we must have

$$t(T, \dots) = t_0(T - T_c) + \dots$$

$$u(T, \dots) = u_0 + u_1(T - T_c) + \dots$$

$$K(T, \dots) = K_0 + K_1(T - T_c) + \dots$$

where $t_0, u_0, K_0 > 0$

To find $\bar{m}$,

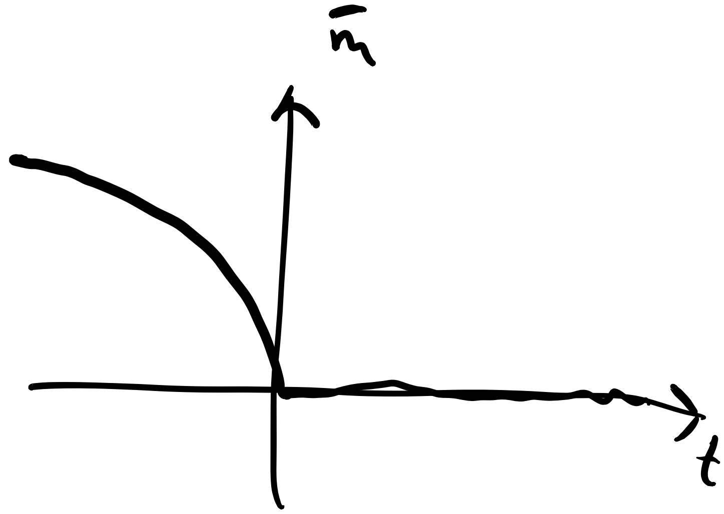
$$\frac{\partial f}{\partial m} \equiv 0 = t\bar{m} + 4u\bar{m}^3 - h$$

$h = 0$ $\quad \bar{m} = \begin{cases} 0 & t > 0 \\ \sqrt{\frac{-t}{4u}} & t < 0 \end{cases}$

$$t\bar{m} + 4u\bar{m}^3 = 0$$

$$\bar{m}(t + 4u\bar{m}^2) = 0$$

$$\bar{m} = 0 \qquad \bar{m} = \sqrt{\frac{-t}{4u}}$$



i.e. $\beta = \frac{1}{2}$.

At $T = T_c$ $(t = 0)$ $\quad \bar{m} = \left(\frac{h}{4u}\right)^{\frac{1}{3}}$

$\Rightarrow \quad \delta = 3$.

Susceptibility - magnetic response (tensor)

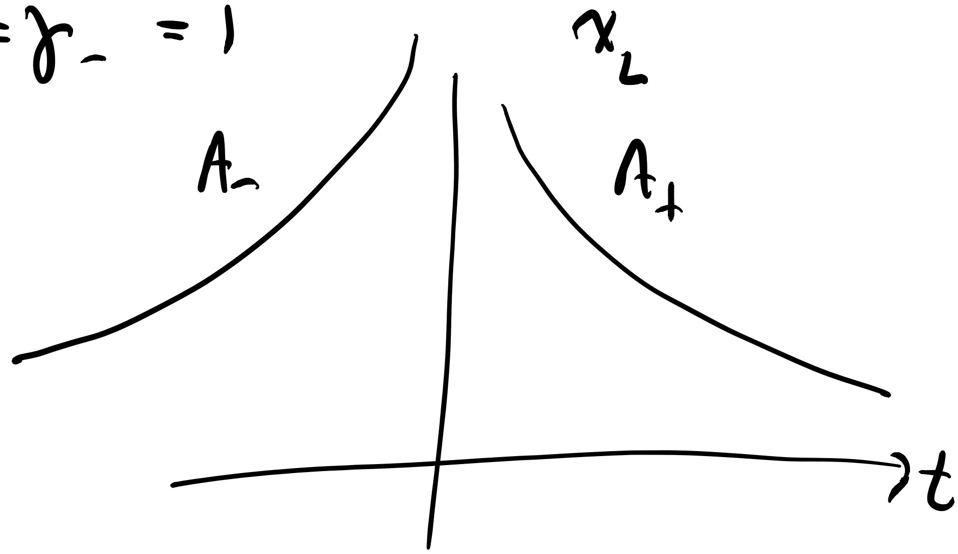
$$\chi_L = \left.\frac{\partial \bar{m}}{\partial h}\right|_{h=0}$$

$\uparrow$
longitudinal

$$\chi_L^{-1} = \left.\frac{\partial h}{\partial \bar{m}}\right|_{h=0} = t + 12u\bar{m}^2$$

$$= \begin{cases} -2t & t<0 \\ t & t>0 \end{cases}$$

$\gamma_+ = \gamma_- = 1$



$\dfrac{A_+}{A_-} = 2$ is <u>universal</u>

<u>Heat capacity</u>

$$f(\bar{m}, h=0) = \left.\frac{\beta H}{V}\right|_{h=0} = \begin{cases} -\dfrac{t^2}{16u} & t<0 \\ 0 & t>0 \end{cases}$$

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \log Z \qquad \frac{\partial}{\partial \beta} = \frac{\partial t}{\partial \beta} \frac{\partial}{\partial t}$$

$$\simeq k_B T_c \frac{\partial}{\partial t} \log Z$$

$T \sim T_c$

$$t = \frac{\frac{1}{k_B \beta} - T_c}{T_c}$$

$$C_{sing} = \frac{1}{V} \frac{\partial \langle E \rangle}{\partial T} \qquad \frac{\partial}{\partial T} = \frac{\partial t}{\partial T} \frac{\partial}{\partial t} = \frac{1}{T_c} \frac{\partial}{\partial t}$$

$$= \frac{k_B}{V} \frac{\partial^2}{\partial t^2} \log Z$$

$$= -k_B \frac{\partial^2 f}{\partial t^2} = k_B \begin{cases} +\frac{1}{8u} & t < 0 \\ 0 & t > 0 \end{cases}$$

here $\frac{\beta F}{V} = f = -\frac{1}{V} \log Z$

$\alpha_+ = \alpha_- = 0$