Fluctuations, Correlations and Susceptibilities

What is the influence of spatial fluctuations around the Landau mean field?

– In particular above d_{LC} → fluctuations do not destroy LRO, but how does it modify our picture?

$$\beta H[\underline{m}(\underline{x})] = \int d^d\underline{x} \; \frac{t}{2} m^2 + u m^4 + \frac{K}{2} (\underline{\nabla}\, \underline{m})^2$$

Recall: most probable state

$$\underline{m}(\underline{x}) = \bar{m}\, \hat{\underline{e}}_1$$ → spontaneous direction

$$\bar{m} = \begin{cases} 0 & t > 0 \\ \sqrt{\frac{-t}{4u}} & t < 0 \end{cases}$$

Small fluctuations

$$\underline{m}(\underline{x}) = [\bar{m} + \underbrace{\varphi_l(\underline{x})}_{\text{longitudinal}}]\, \hat{\underline{e}}_1 + \sum_{\alpha=2}^{n} \underbrace{\varphi_t^{\alpha}}_{\text{transverse}}\, \hat{\underline{e}}_\alpha$$

Let us expand to second order (i.e. we consider Gaussian fluctuations)

$$|\underline{\nabla}\,\underline{m}|^2 = |\underline{\nabla}\varphi_l|^2 + |\underline{\nabla}\varphi_t|^2$$

$$|\underline{m}|^3 = \bar{m}^2 + 2\bar{m}\varphi_l + \varphi_l^2 + \varphi_t^2$$

$$|\underline{m}|^4 = \bar{m}^4 + 4\bar{m}^3\varphi_l + 6\bar{m}^2\varphi_l^2 + 2\bar{m}^2\varphi_t^2 + O(\varphi^3)$$

$$\beta H = V\left(\frac{t}{2}\bar{m}^2 + u\bar{m}^4\right) \qquad f$$

$$+\int d^d\underline{x}\ \frac{K}{2}|\underline{\nabla}\varphi_l|^2 + \frac{1}{2}(t + 12u\bar{m}^2)\varphi_l^2$$

$$+\int d^d\underline{x}\ \frac{K}{2}|\underline{\nabla}\varphi_t|^2 + \frac{1}{2}(t + 4\bar{m}^2u)\varphi_t^2$$

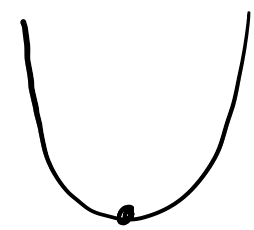
define length scales

$$\frac{K}{\xi_l^2} \equiv t + 12u\bar{m}^2 = \begin{cases} t & t>0 \\ -2t & t<0 \end{cases}$$

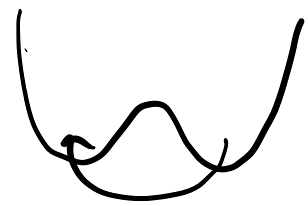
$$\frac{K}{\xi_t^2} \equiv t + 4u\bar{m}^2 = \begin{cases} t & t>0 \\ 0 & t<0 \end{cases}$$

Notes

1. For $t>0$, φ_t and φ_l are the same.



2. For $t<0$, no restoring force to transverse fluctuations (G.M.)



3. At 2^{nd} order, φ_l and φ_t modes are decoupled.

Correlation functions

$$C_{\alpha,\beta}(\underline{q},\underline{q}') \equiv \langle \varphi_\alpha(\underline{q})\, \varphi_\beta(\underline{q}') \rangle_c$$

$\uparrow$

l, t

$$= \int d^d\underline{x}\; e^{i\underline{q}\cdot\underline{x}} \int d^d\underline{x}'\; e^{i\underline{q}'\cdot\underline{x}'}\; \underbrace{\langle \varphi_\alpha(\underline{x})\,\varphi_\beta(\underline{x}')\rangle_c}_{=\,G_{\alpha\beta}(\underline{x},\underline{x}')}$$

Let $\bar{\underline{x}} = \frac{1}{2}(\underline{x}+\underline{x}')$ C.o.M.

$$\underline{y} = \underline{x}-\underline{x}'$$

Translation invariance $G_{\alpha\beta}(\underline{x}-\underline{x}') = G_{\alpha\beta}(\underline{y})$

$$\rightsquigarrow C_{\alpha\beta}(\underline{q},\underline{q}') = \int d^d\underline{X}\; e^{i(\underline{q}+\underline{q}')\cdot\underline{X}} \int d^d\underline{y}\; e^{i(\underline{q}-\underline{q}')\cdot\frac{\underline{y}}{2}}\, G_{\alpha\beta}(\underline{y})$$

$$= (2\pi)^d\,\delta^d(\underline{q}+\underline{q}')\; G_{\alpha\beta}(\underline{q})$$

At Gaussian order since modes are decoupled

$$G_{\alpha\beta}(\underline{q}) = G_\alpha(\underline{q})\,\delta_{\alpha\beta}$$

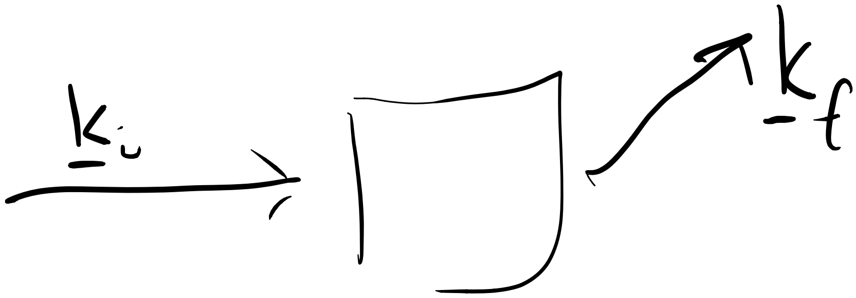
<u>Key results</u>

$$G_{\alpha\beta}(\underline{q},\underline{q}') = (2\pi)^d\,\delta^d(\underline{q}+\underline{q}')\,\delta_{\alpha\beta}\,G_\alpha(\underline{q})$$

$$Z = \int \mathcal{D}\varphi_\alpha(\underline{x}) \exp\left\{-\frac{1}{2}\sum_\alpha \int d^d\underline{x}\left(\frac{\varphi_\alpha^2}{\xi_\alpha^2} + |\underline{\nabla}\varphi_\alpha|^2\right)\right\}$$

$$\left(-\nabla'^2 + \xi_\alpha^{-2}\right) G_\alpha(\underline{x}-\underline{x}') = \delta^d(\underline{x}-\underline{x}')$$

$$G_\alpha(\underline{q}) = \frac{1}{q^2+\xi_\alpha^{-2}}$$



$\underline{k}_f - \underline{k}_i = \underline{q}$

Scattering – Ornstein-Zernike

Amplitude $\quad A(\underline{q}) \propto \langle \underline{k}_f | u | \underline{k}_i \rangle$

$$\propto \int d^d\underline{x} \; e^{i\underline{q}\cdot\underline{x}} \; \underline{m}(\underline{x})$$

Born approx.

So form factor / scattering probability

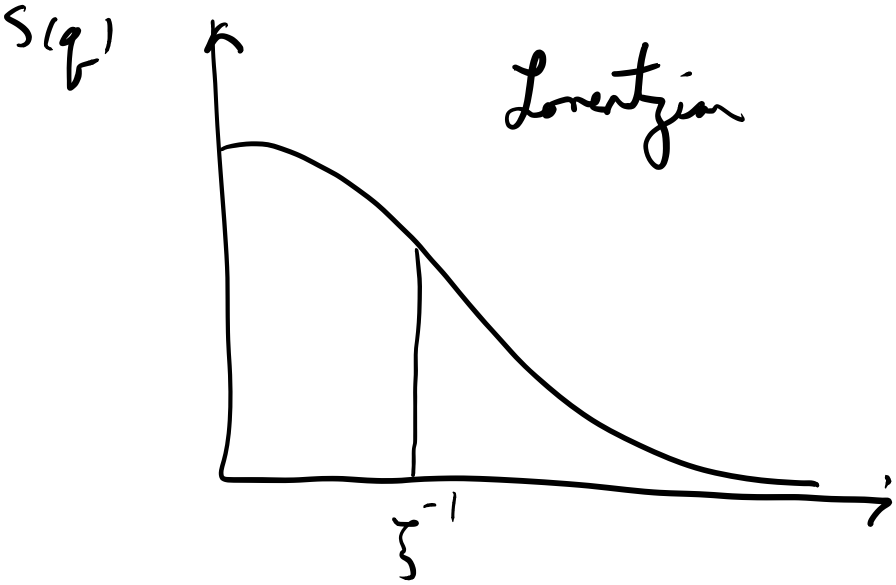
$$S(\underline{q}) = \langle |A(\underline{q})|^2 \rangle$$

$$\propto \langle |m(\underline{q})|^2 \rangle$$

$$S_{l,t}(\underline{q}) \propto \langle |q_{l,t}(\underline{q})|^2 \rangle + \bar{m}^2 \, \delta^d(\underline{q})$$

spontaneous magnetisation
ordered component

$$\frac{1}{k\left(q^2 + \xi_{l,t}^{-2}\right)}$$



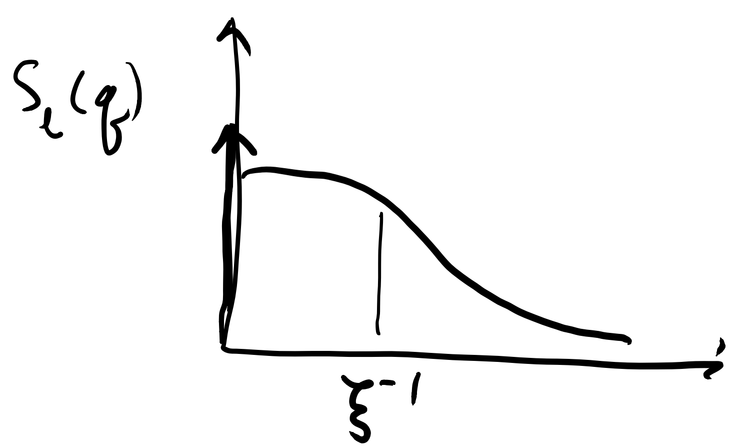
Lorentzian

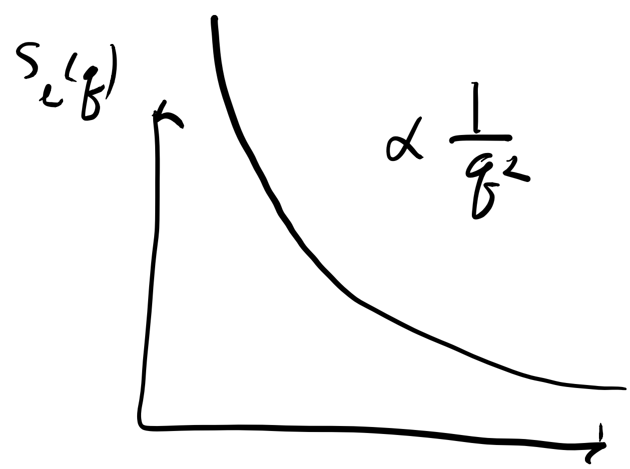
$$\xi^{-1} = \sqrt{\frac{t}{k}} \quad \leftarrow \text{narrows as } t \to 0^+$$

$$S(q) \to \frac{1}{q^2} \quad \text{as } t \to 0^+$$

- Origin of critical opalescence

$t < 0$





$\propto \frac{1}{q^2}$

G.M.

Experimentally $\left. S(q)\right|_{t=0} \sim \frac{1}{q^{2-\eta}}$

new universal exponent

$\eta = 0$ under Gaussian approx.

Real space correlation function

$$\langle m_\alpha(\underline{x}) - \bar{m}_\alpha \rangle = 0$$

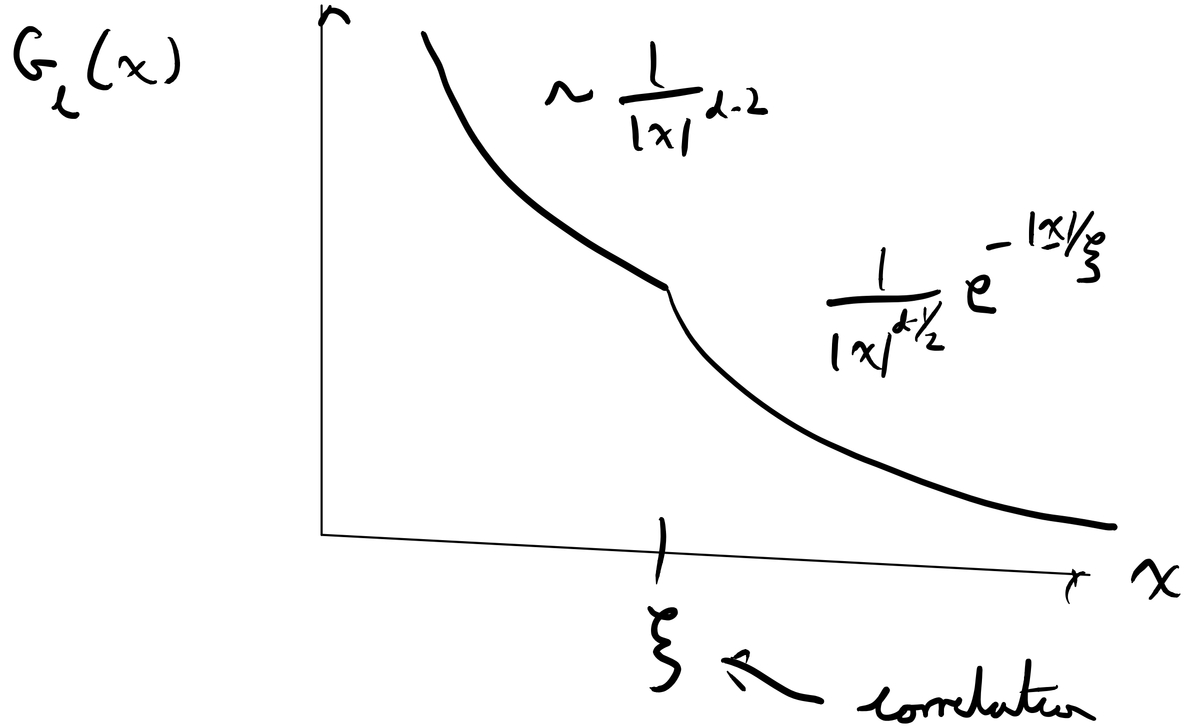
$$\langle (m_\alpha(\underline{x}) - \bar{m}_\alpha)(m_\beta(\underline{x}') - \bar{m}_\beta) \rangle \equiv \langle m_\alpha(\underline{x})\, m_\beta(\underline{x}') \rangle_c$$

$$= \langle \varphi_\alpha(\underline{x})\, \varphi_\beta(\underline{x}') \rangle = G_{\alpha\beta}(\underline{x}, \underline{x}')$$

$$= -\frac{1}{K} \delta_{\alpha\beta} I_d(\underline{x} - \underline{x}', \xi_\alpha)$$

where $I_d(x, \xi) = -\int \frac{d^d q}{(2\pi)^d} \frac{e^{i\underline{q}\cdot\underline{x}}}{q^2 + \xi^{-2}}$ $\qquad q \sim \frac{1}{x} \gg \frac{1}{\xi}$

$$\simeq \begin{cases} c_d = \dfrac{|\underline{x}|^{2-d}}{(2-d)\, S_d} & x \ll \xi \\[2ex] \dfrac{\xi^{(3-d)/2}}{(2-d)\, S_d\, |x|^{(d-1)/2}} e^{-|x|/\xi} & x \gg \xi \end{cases}$$



$$\xi_L = \begin{cases} \sqrt{\dfrac{K}{t}} & t > 0 \\ \sqrt{\left(\dfrac{-K}{2t}\right)} & t < 0 \end{cases}$$

$$\xi_\pm = \xi_0 \, B_\pm \, |t|^{-\nu_\pm} \qquad \nu_\pm = \frac{1}{2}$$

$$\frac{B_+}{B_-} = \sqrt{2} \quad \leftarrow \text{Universal}$$

$$\xi_t = \begin{cases} \xi_L & t > 0 \\ \infty & t < 0 \end{cases}$$

G.M.

Susceptibilities

$$\chi_L \sim \int d^d\underline{x}\; G_L(\underline{x}) \sim \int_0^{\xi_L} \frac{d^d\underline{x}}{x^{d-2}}$$

$$\sim \xi_L^2 = A_\pm |t|^{-1}$$

χ_t is the same

But $t<0$

$$\chi_t \sim \int d^d\underline{x}\; G_t(\underline{x},0)$$

$$\sim \int_0^L \frac{d^d\underline{x}}{x^{d-2}} \sim L^2$$