Comment on "Magnetic-Field-Tuned Quantum Phase Transition in the Insulating Regime of Ultrathin Amorphous Bi Films"

A recent Letter by Lin and Goldman [1] presented experimental data for the relative magnetoresistance (MR) in disordered thin films, which were interpreted as evidence of a quantum phase transition. Such films are known to exhibit a superconductor (SC)-insulator transition as a function of disorder [2], and a huge peak in the resistance $R(B)$ with magnetic field B [3,4]. These highly disordered samples were insulating at zero B. The experimental results supporting the quantum phase transition scenario are: (a) the relative magnetoresistance, $MR(B, B_0) = [R(B) - R(B_0)]/R(B_0)$, at $B_0 = 0$ was temperature (T) independent at a specific, nonuniversal, field B_C, and (b) near this point all the different-T curves collapsed upon rescaling $R = R_C F(|B - B_C|/T^{1/\nu z})$, where ν and z were interpreted as the critical exponents of the transition. In this Comment, we present an alternative interpretation based on activated transport in a disordered landscape. We first present numerical simulations, and then support them by simple analytic arguments.

Our numerical simulations were performed using a new *ab initio* technique, based on the disordered negative-U Hubbard model, that fully captures the effects of thermal phase fluctuations [5]. The results of this method describe the observed phenomenology of transport through thin disordered SC films, including the origin of the magnetoresistance peak [6]. Here we report results for more disordered systems, which, as in the experiment, are resistive at zero B (we used an on site energy standard deviation of $W = 6t$, where t is the lattice hopping integral, on site interaction $U = 1.6t$, and 0.37 filling). The inset of Fig. 1(b) depicts $R(B)$ for several temperatures, with the resulting MR shown in Fig. 1(a), where the main experimental result is reproduced—following a peak, the MR isotherms cross at a constant magnetic field. Near that point, all the curves collapse [Fig. 1(b)], using the same scaling analysis as in [1], with $\nu z = 0.89$. The sample displays no notable phenomenon in the local currents and chemical potential at B_C.

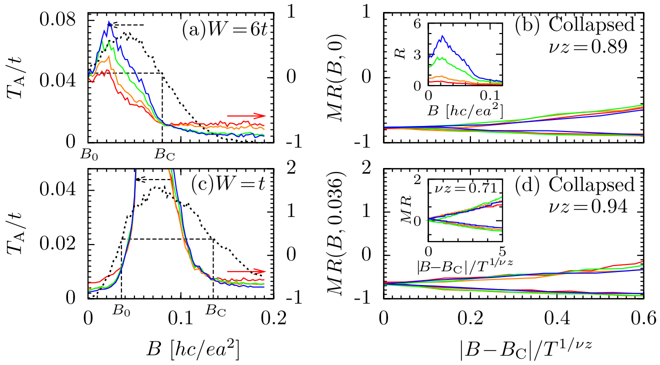

FIG. 1 (color online). (a), (c) The MR curves (solid lines) and activation temperature (dotted lines) with magnetic field. The upper plots were taken at disorder $W = 6t$ and the lower at $W = t$. The blue curve (lowest at large fields) is at low temperature $T = 0.02t$, and red (highest at large fields) high temperature $T = 0.07t$. (b), (d) The MR with scaled magnetic field. The inset (b) shows the variation of resistance with magnetic field and (d) the MR for the experimental data from Ref. [3].

Since our numerical calculations neglect quantum fluctuations, the source of our crossing point B_C cannot be the putative quantum phase transition [1]. To understand the crossing we note that both in the theory and in the experiment, the resistance is activated, $R(B, T) = R_0(B)e^{T_A(B)/T}$, with $T_A(B)$ the activation temperature at field B, and $R_0(B) \approx h/4e^2$ is the high temperature resistance. Figure 1(a) shows that $T_A(B)$, in agreement with experiment, is a nonmonotonic function, and, in fact, B_C corresponds to $T_A(B_C) = T_A(0)$. If $R(B, T)$ obeys the activated behavior above, $MR(B, 0)$ becomes T independent at $B = B_C$. Moreover, expanding $T_A(B)$ around $B = B_C$, we find that the scaling function

$$MR(B, B_0) = \frac{R_0(B)}{R_0(B_0)}\left(1 + \frac{T_A'(B_C)(B - B_C)}{T}\right) - 1, \quad (1)$$

is in agreement with the experimental fitted form with $\nu z = 1$. (The deviations from perfect scaling come from the weak dependence of $R_0(B)$ on B, and from the deviations, both experimentally and numerically, from simple activation at lower temperatures.)

If our interpretation is correct, and B_C was only determined by $T_A(B_C) = T_A(0)$, the same behavior should be observed in less disordered samples for $MR(B, B_0)$, where $T_A(B_C) = T_A(B_0)$ and $B_0 > 0$. Indeed in Figs. 1(c) and 1(d) we present results for a sample with lower disorder $W = t$ that is SC at $B = 0$. With $\nu z = 0.94$ the MR isotherms all cross at $B = B_C$, with a reasonable collapse. Moreover, the inset of Fig. 1(d) depicts the excellent collapse of the experimental data published in Ref. [3] for a lower disorder sample, with $B_0 = 4$ T, $B_C = 12.8$ T, and $\nu z = 0.71$, supporting our scenario.

In summary, using *ab initio* simulations and analytic arguments, we have demonstrated an alternative explanation of the experimental results of Ref. [1]. The crossing of the MR curves can be understood entirely in terms of activated transport, which our previous analysis attributed to transport through Coulomb blockade islands [6]. Finally, we have made a specific prediction to test our analysis.

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