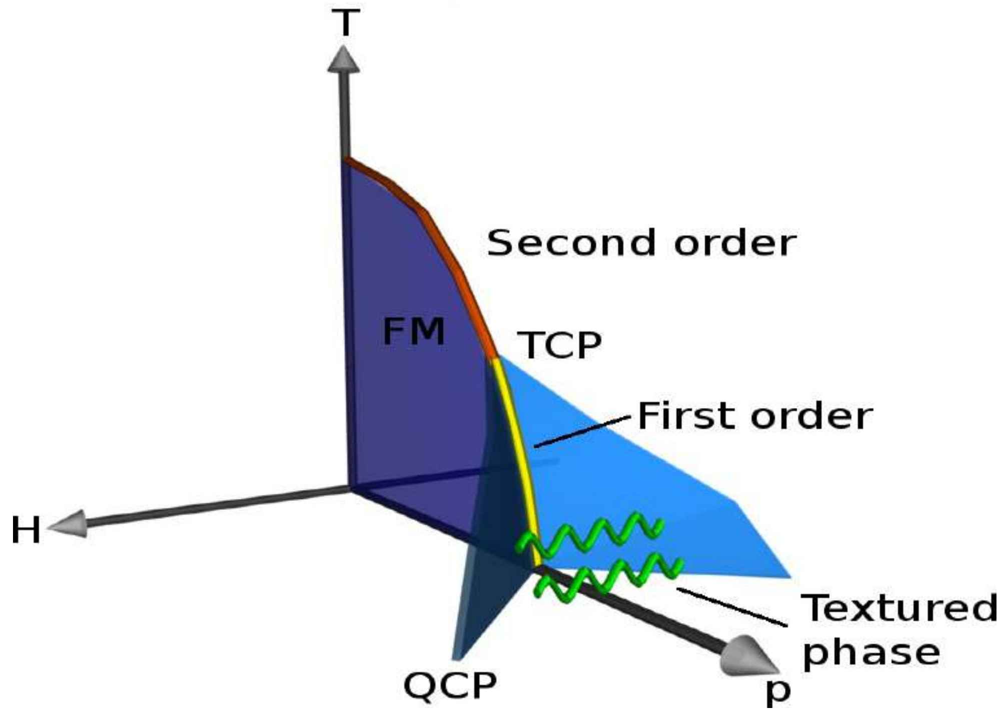
Inhomogeneous phase formation on the border of itinerant ferromagnetism



Gareth Conduit[1, 2], Andrew Green[3] & Ben Simons[4]

1. Weizmann Institute, 2. Ben Gurion University, 3. University of St. Andrews, 4. University of Cambridge

G.J. Conduit & B.D. Simons, Phys. Rev. A **79**, 053606 (2009)

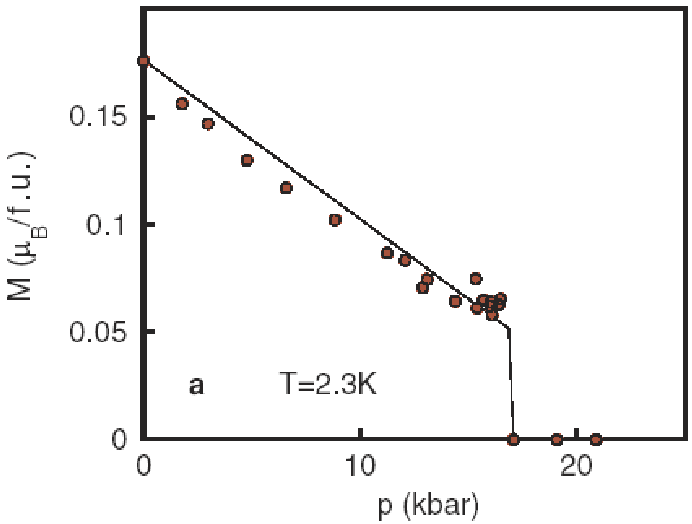
G.J. Conduit, A.G. Green & B.D. Simons, Phys. Rev. Lett. **103**, 207201 (2009)

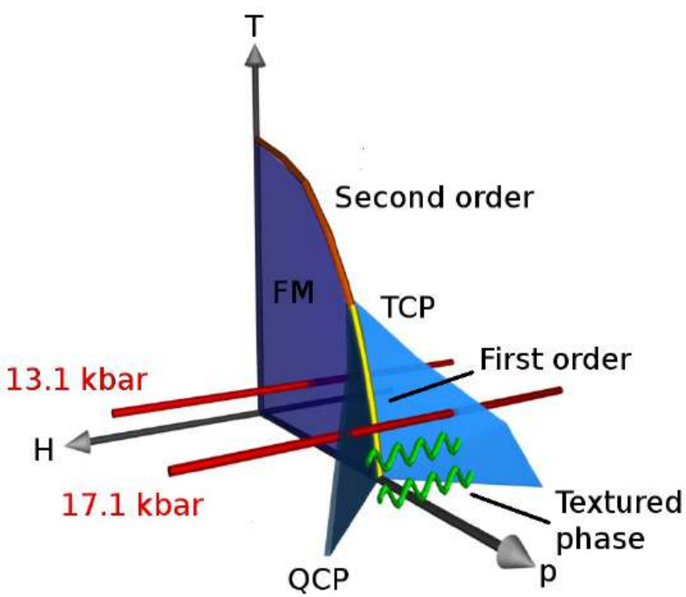
G.J. Conduit & B.D. Simons, Phys. Rev. Lett. **103**, 200403 (2009)

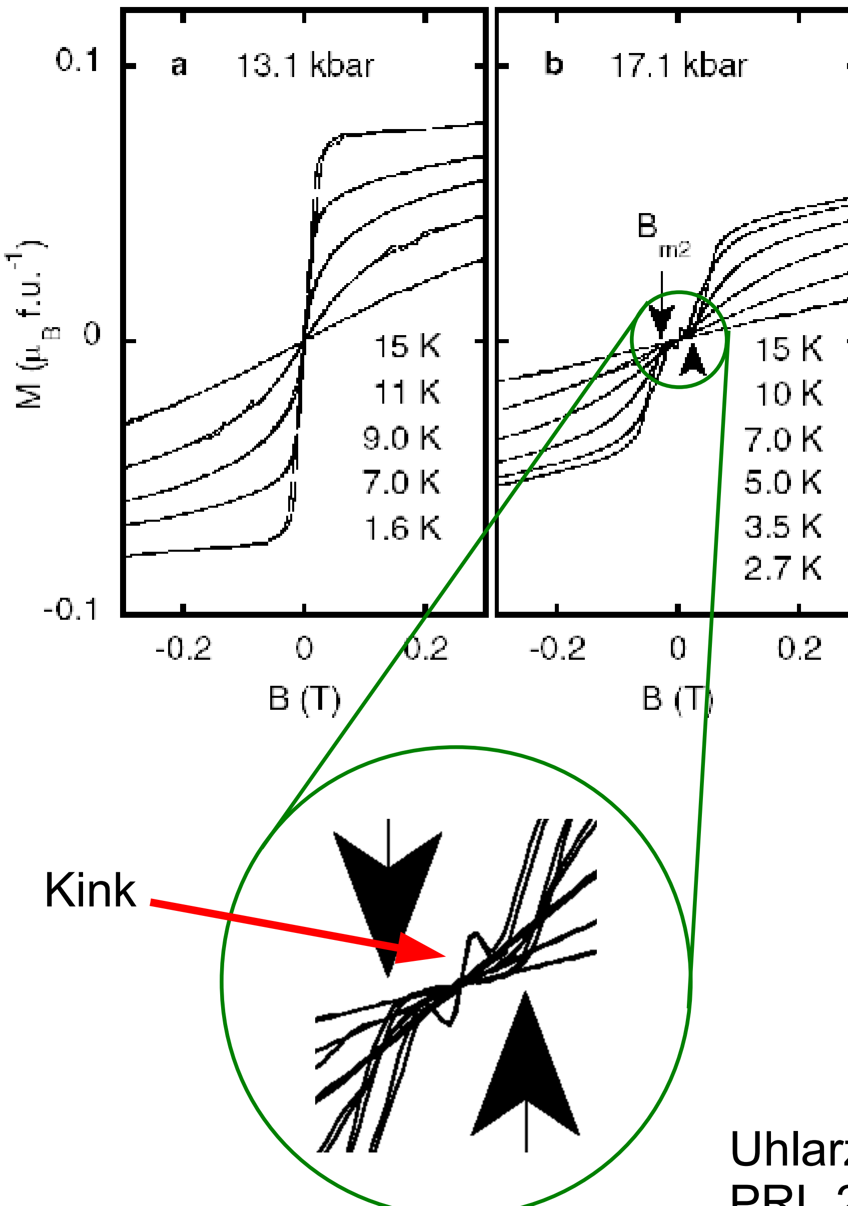
G.J Conduit & E. Altman, arXiv: 0911.2839

Breakdown of Stoner criterion — $ZrZn_2$

- At low temperature and high pressure $ZrZn_2$ has a first order transition



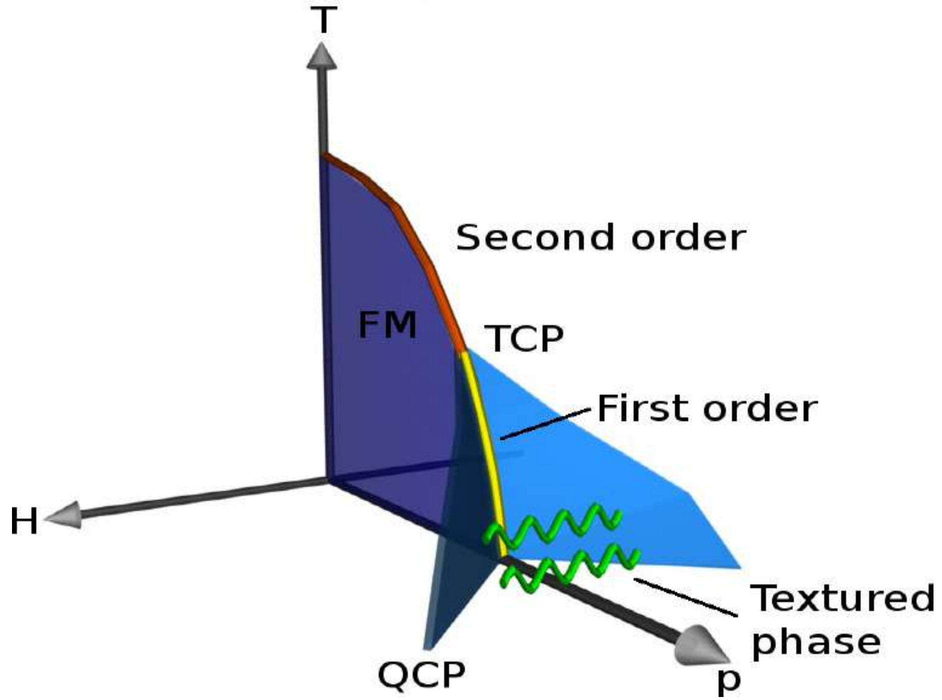




Uhlarz *et al.*, PRL 2004

Breakdown of Stoner criterion

- Generic phase diagram of the itinerant ferromagnet



FM: Fully magnetised
TCP: Tricritical point
QCP: Quantum critical point

- Two explanations of first order phase behaviour:

(1) Lattice-driven peak in the density of states
(Pfleiderer *et al*. PRL 2002, Sandeman *et al*. PRL 2003)

(2) Transverse quantum fluctuations (Belitz *et al*. Z. Phys. B 1997)

Fluctuation corrections

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\boldsymbol{r} \sum_{\sigma} \bar{\psi}_{\sigma}(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_{\sigma}-g\,\bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Integrate over fluctuations in magnetization and density and develop a perturbation theory in interaction strength

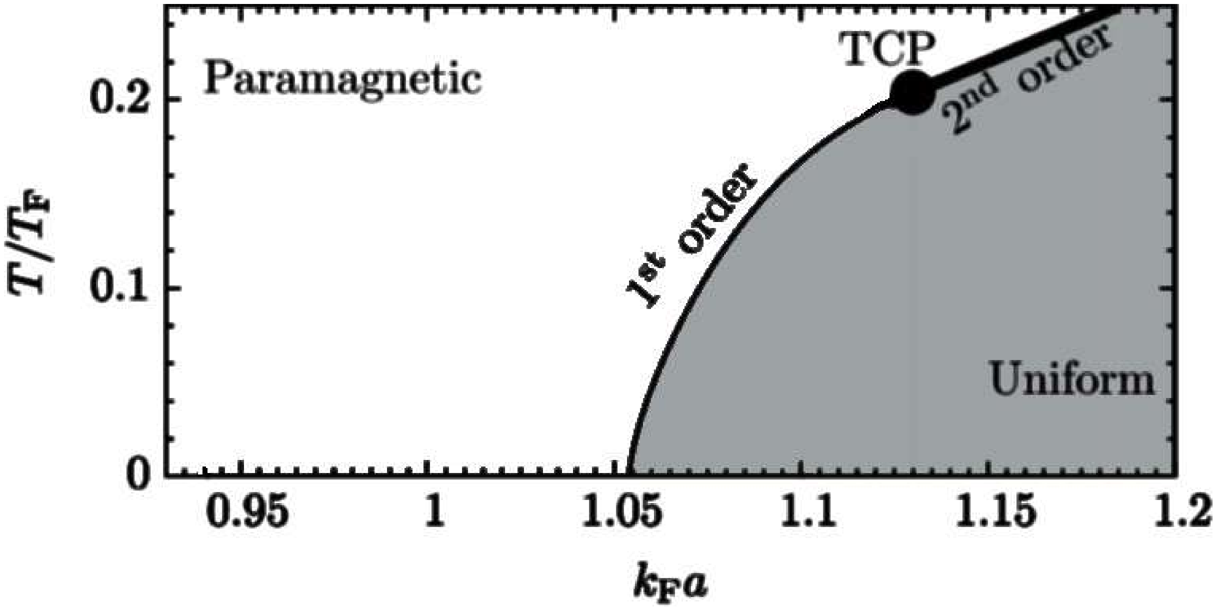
$$F=F_0+\frac{1-g\nu}{2\nu}m^2+um^4+vm^6+g^2\left(r\,m^2+w\,m^4\ln|m|\right) \qquad k_{\mathrm{F}}a_{\mathrm{crit}}=1.05$$

- First order transition[1]

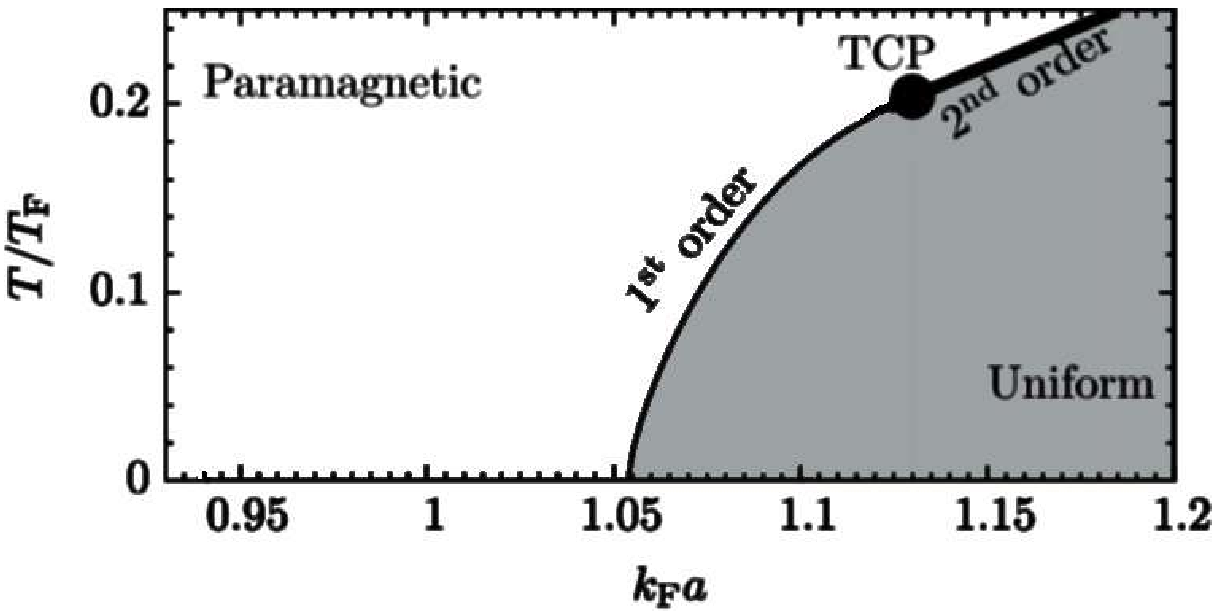
[1]Abrikosov (1958), Duine & MacDonald (2005), Belitz *et al*. Z. Phys. B (1997) & Conduit & Simons (2009)

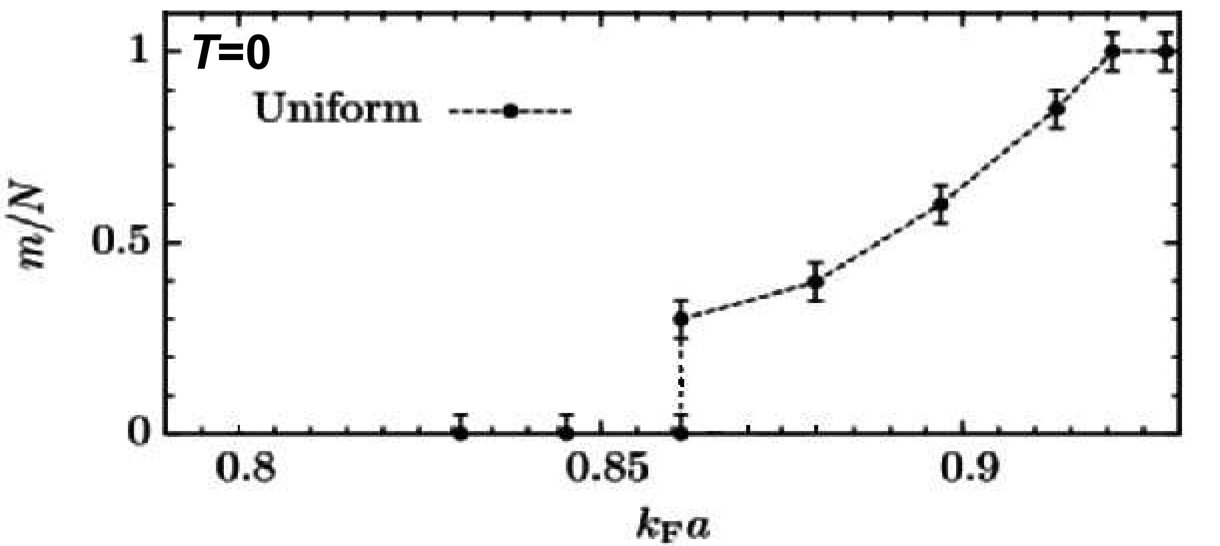
Results

- First order ferromagnetic phase transition



Quantum Monte Carlo verification





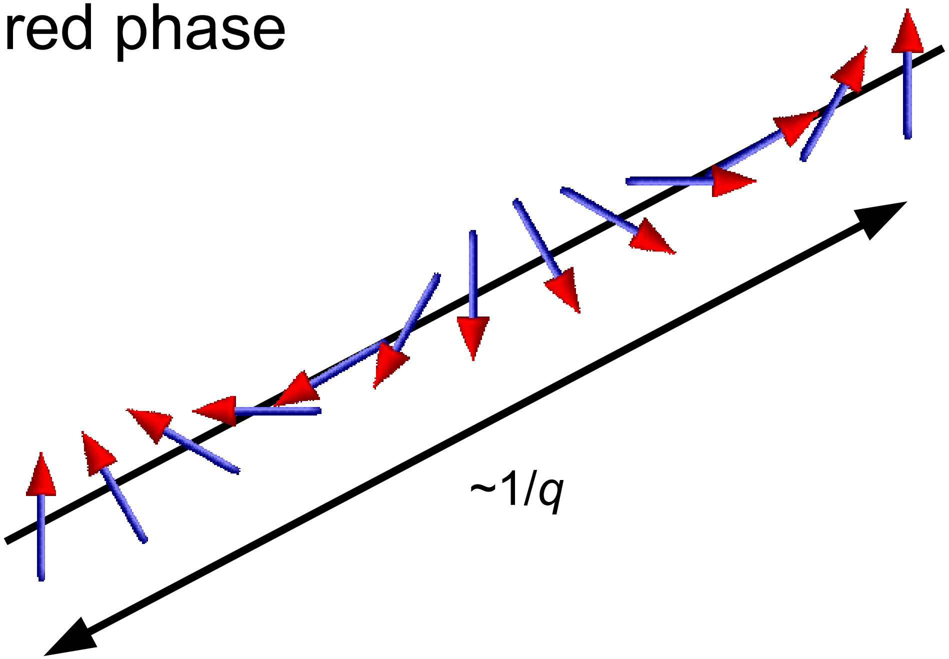
Textured order parameter

$$Z=\int D\psi \exp\left(-\iint d\tau\, d\mathbf{r} \sum_\sigma \bar{\psi}_\sigma(-i\hat{\omega}+\hat{\epsilon}-\mu)\psi_\sigma - g\,\bar{\psi}_\uparrow\bar{\psi}_\downarrow\psi_\downarrow\psi_\uparrow\right)$$

- Gauge transformation brings textured phase to uniform order parameter
- Coefficient of m^4 has the same form as q^2m^2

$$F=F_0+rm^2+um^4+vm^6+\frac{u k_F^2}{24\pi^2 a^2}q^2 m^2$$

- Tricritical point accompanied by textured phase

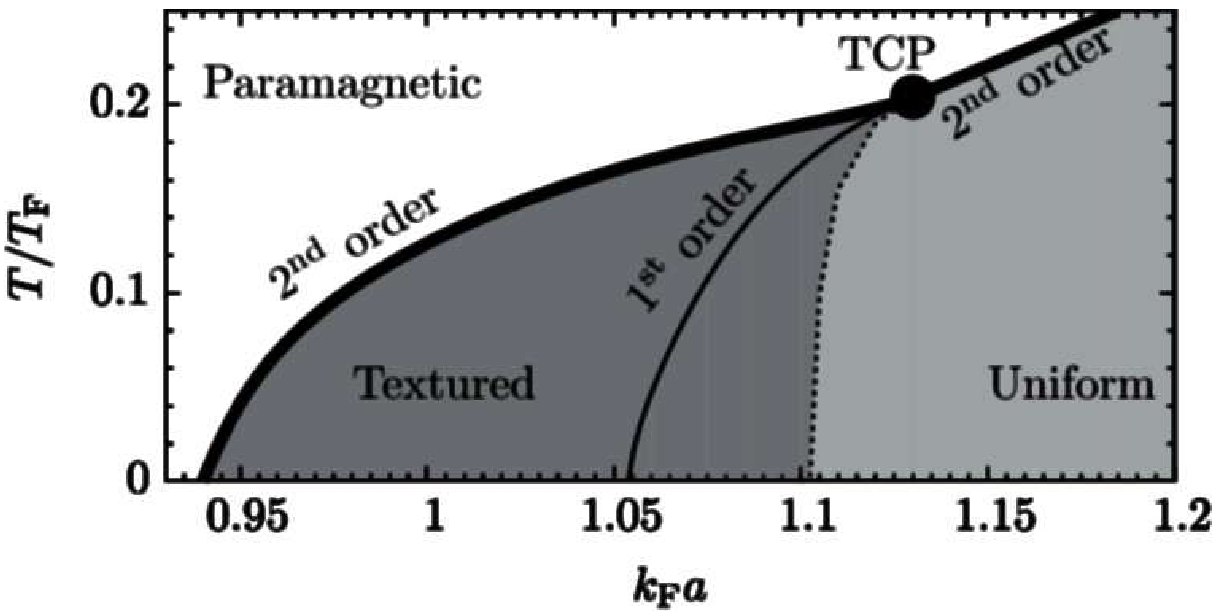


Belitz *et al*. Rev. Mod. Phys. (2005)

Betouras *et al*. PRB (2005)

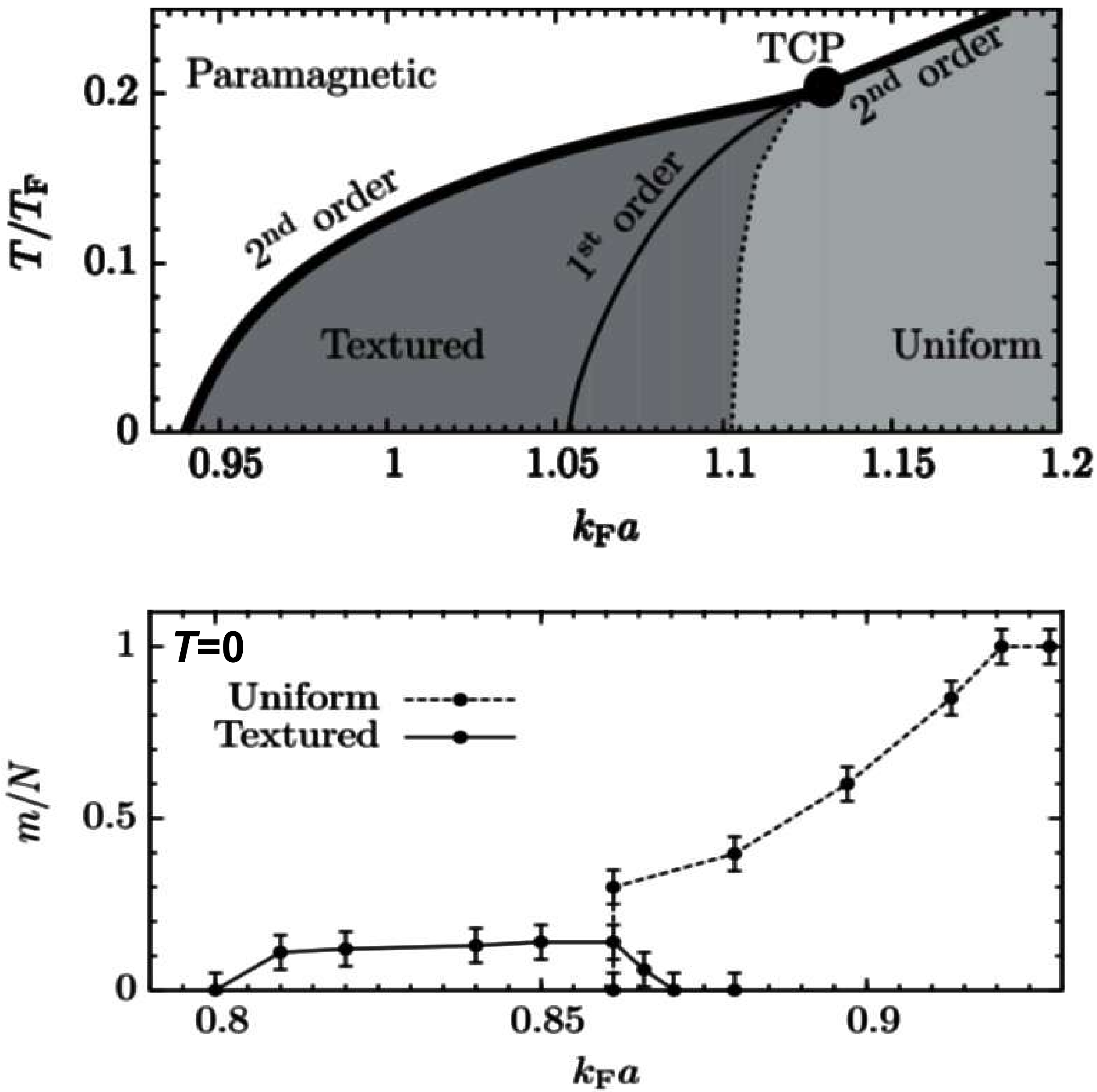
Results

- Textured phase preempts transition



Quantum Monte Carlo: textured phase

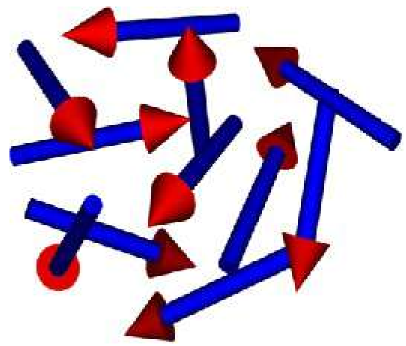
- QMC verifies presence of textured phase



Summary

- Soft transverse magnetic fluctuations drive the ferromagnetic transition first order
- Textured phase preempts ferromagnetic transition
- Verification with Quantum Monte Carlo
- First observation of itinerant ferromagnetism in ultracold atom gases [Jo *et al*. Science **325**, 1521 (2009), Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)] – see session Y31

Condensation of topological defects

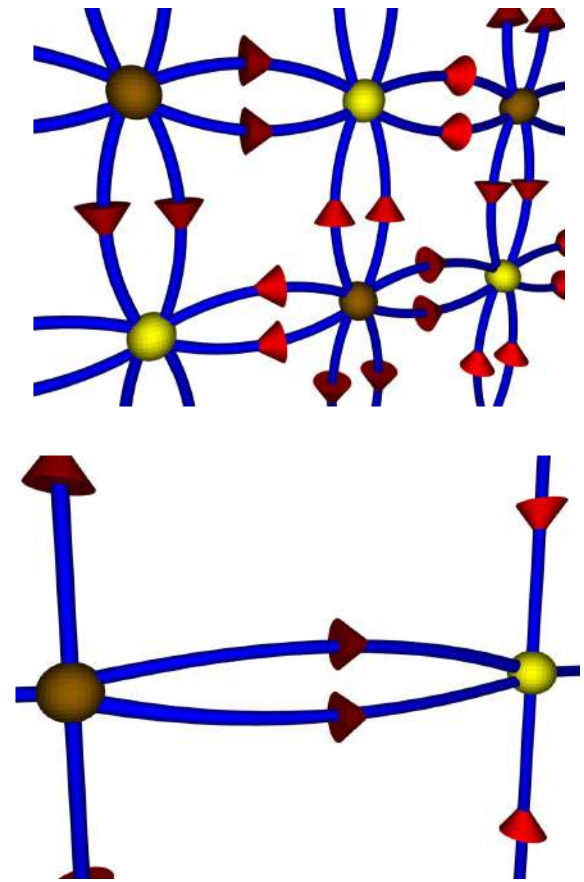


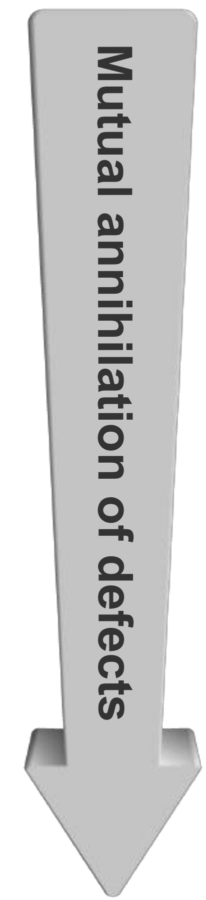
Start in paramagnetic state and ramp up interaction strength

- Defects freeze out from disordered state

- Defect annihilation hinders the formation of the ferromagnetic phase thus raising the required interaction strength

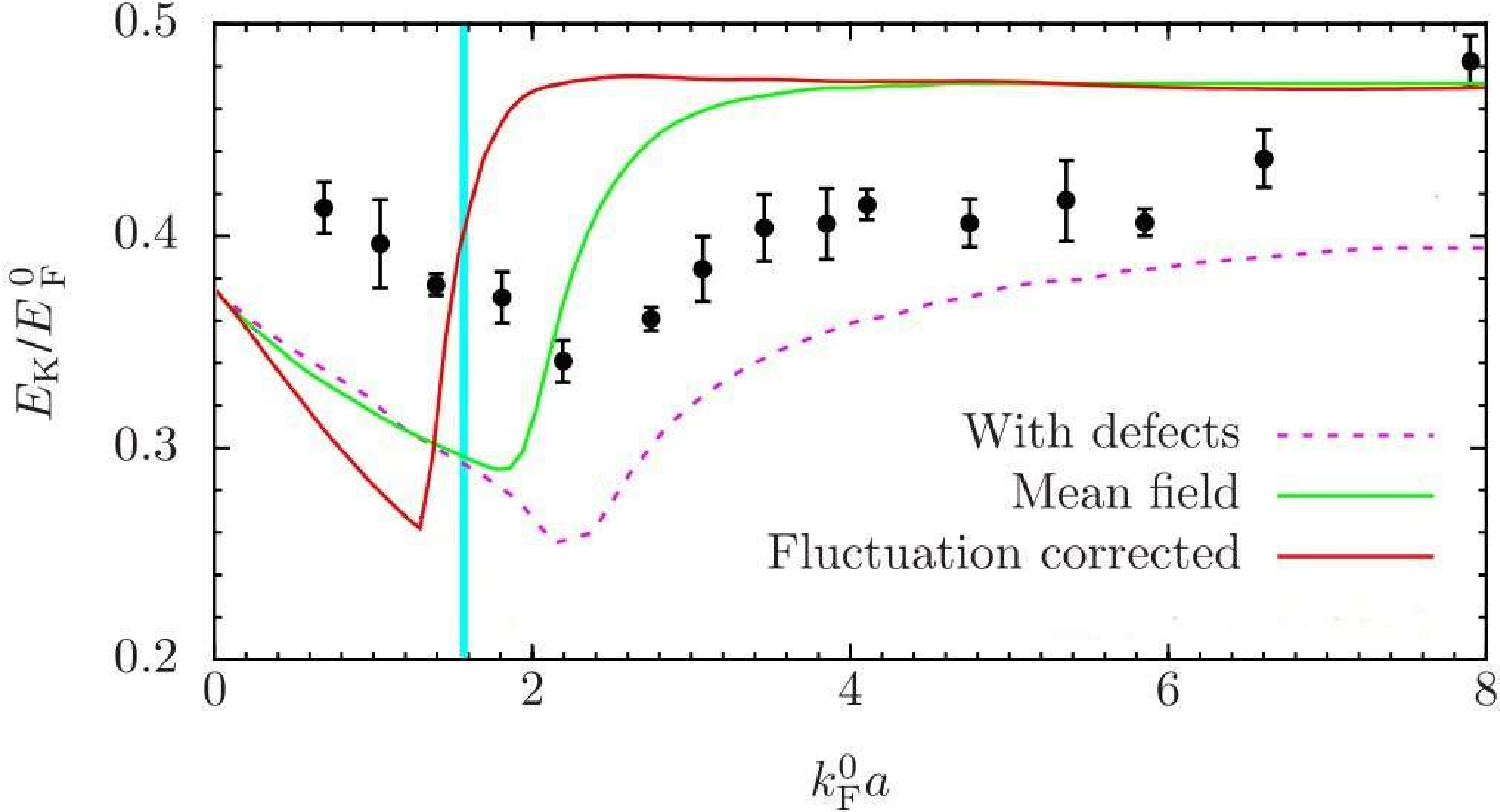
- Defect radius $L \sim t^{1/2}$ [Bray, Adv. Phys. **43**, 357 (1994)]





Condensation of topological defects

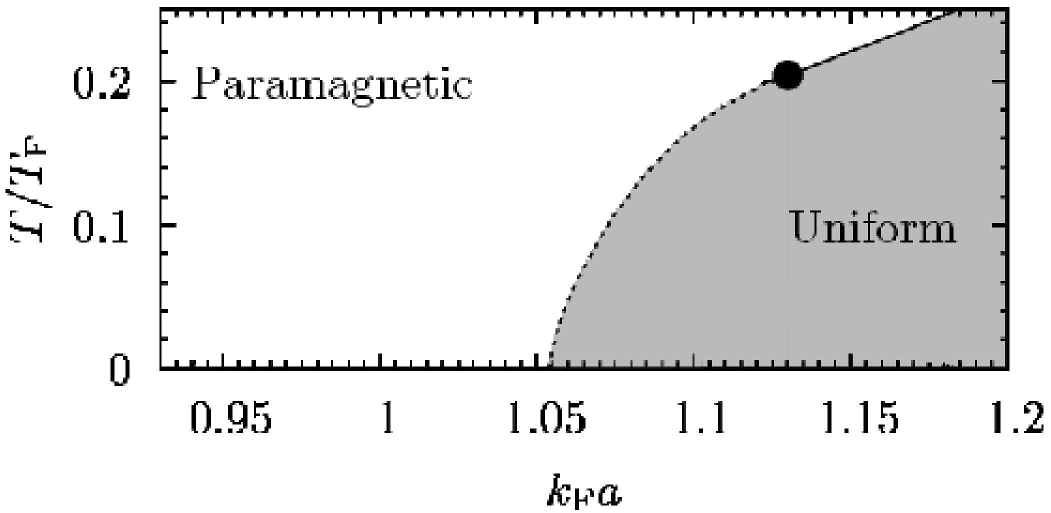
- Condensation of defects inhibits the transition



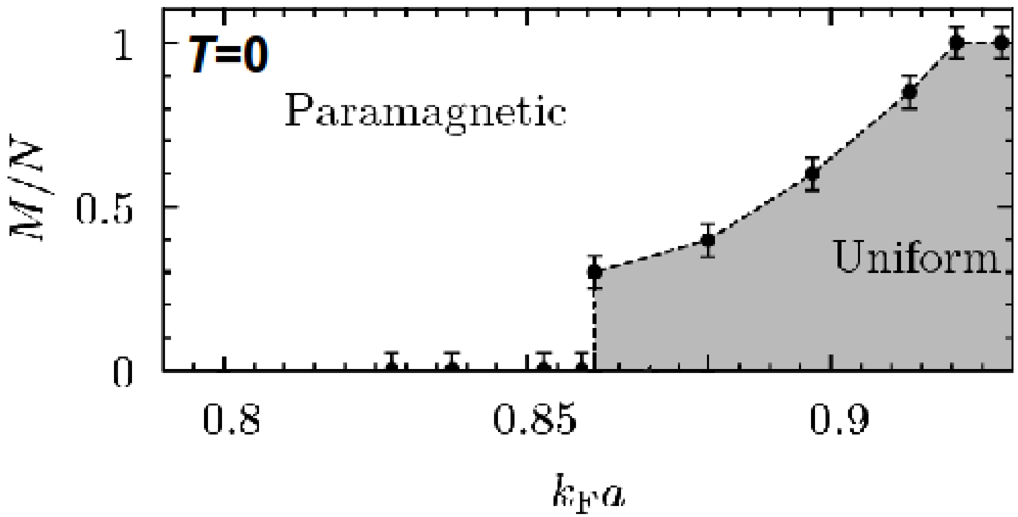
Conduit & Simons, Phys. Rev. Lett. **103**, 200403 (2009)

First order phase transition and Quantum Monte Carlo verification

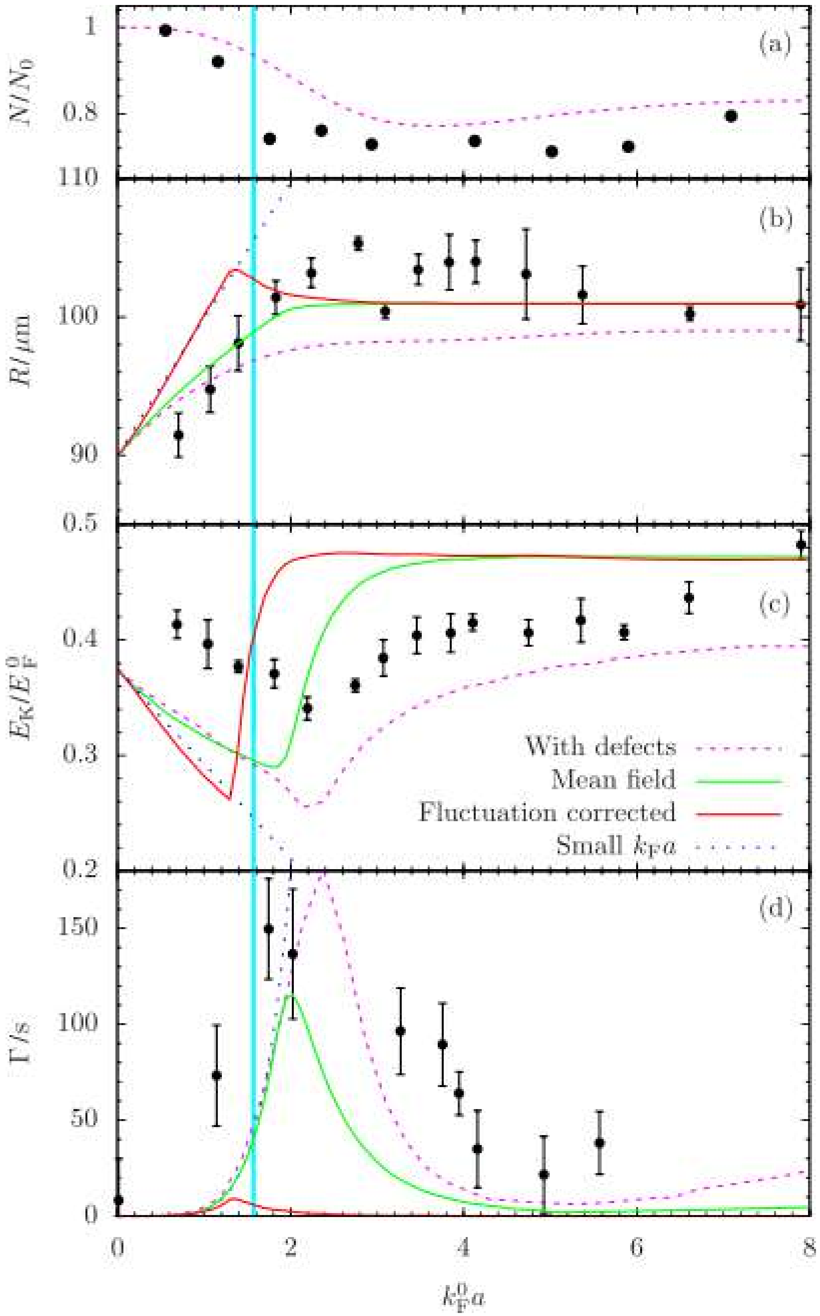
- First order transition into uniform phase with TCP



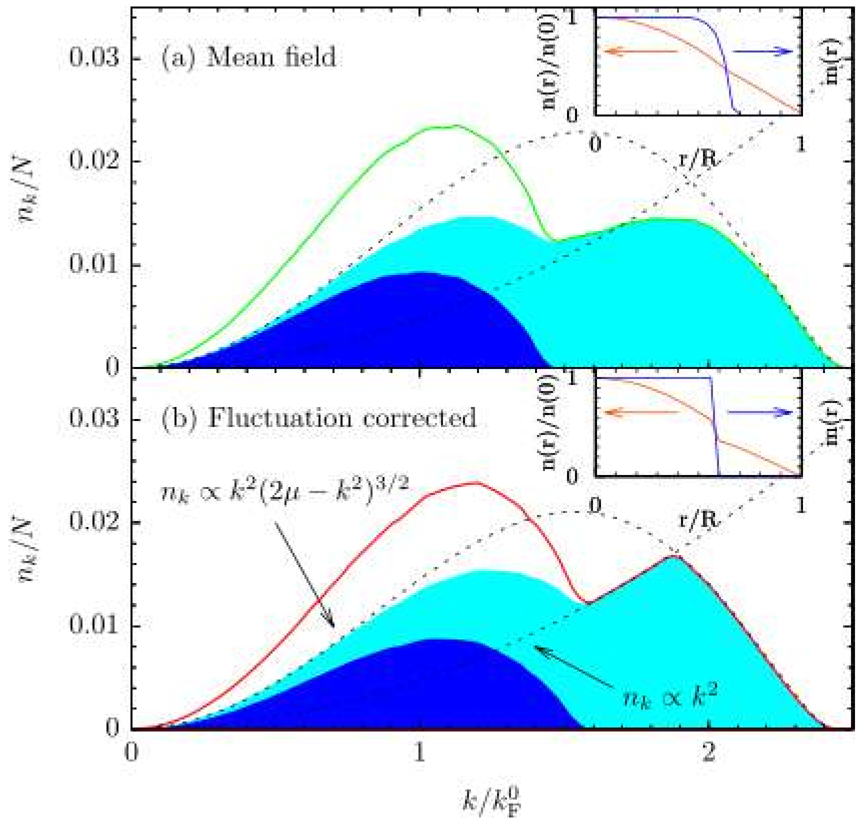
- QMC also sees first order transition



Summary of equilibrium results



Momentum distribution



New approach to fluctuation corrections

$$Z=\int D\psi \exp\left(-\int \sum_{\sigma} \bar{\psi}_{\sigma}(-i\omega+\epsilon-\mu)\psi_{\sigma}-g\int \bar{\psi}_{\uparrow}\bar{\psi}_{\downarrow}\psi_{\downarrow}\psi_{\uparrow}\right)$$

- Analytic strategy:

  1) Decouple in both the density and spin channels (previous approaches employ only spin)

  2) Integrate out electrons

  3) Expand about uniform magnetisation

  4) Expand density and magnetisation fluctuations to second order

  5) Integrate out density and magnetisation fluctuations
- Aim to uncover connection to second order perturbation theory
- Unravel origin of logarithmic divergence in energy and relation to tricritical point structure

Analytical method

- System free energy $F=-k_B T \ln Z$ is found via the partition function

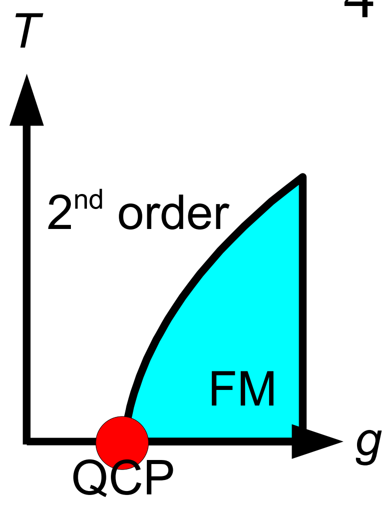
$$Z = \int D\psi \exp\left(-\int \sum_\sigma \bar{\psi}_\sigma (-i\omega + \epsilon - \mu)\psi_\sigma - g\int \bar{\psi}_\uparrow \bar{\psi}_\downarrow \psi_\downarrow \psi_\uparrow\right)$$

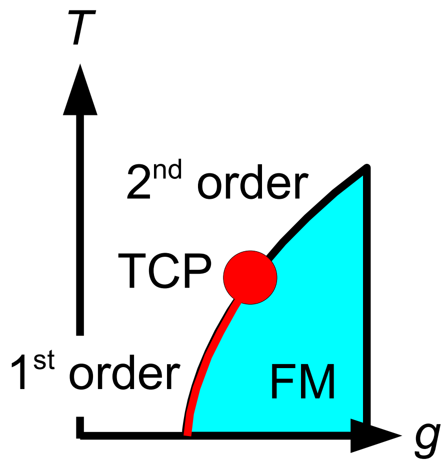
- Decouple using only the average magnetisation $m = \bar{\psi}_\downarrow \psi_\uparrow - \bar{\psi}_\uparrow \psi_\downarrow$

  gives $F \propto (1 - g\nu) m^2$ i.e. the Stoner criterion

- Belitz-Kirkpatrick-Vojta (soft particle-hole & magnetisation) [Belitz *et al.* Z. Phys. B 1997]

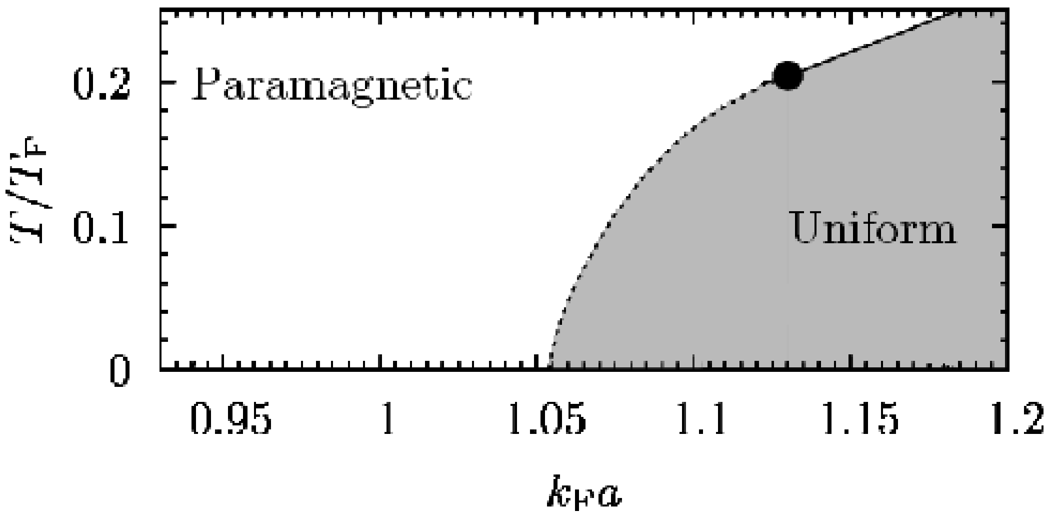
$$F = \frac{1}{2}\left(|\omega|/\Gamma_q + r + q^2\right) m^2 + \frac{u}{4} m^4 \ln(m^2 + T^2) + \cdots - hm$$



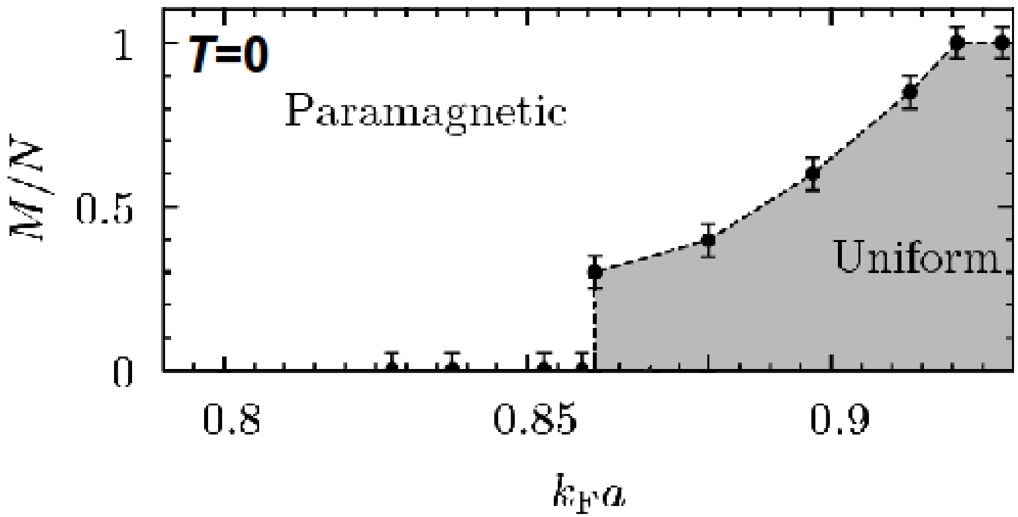


Quantum Monte Carlo verification

- First order transition into uniform phase with TCP



- QMC also sees first order transition



Cold atomic gases — spin

- Two fermionic atom species have a *pseudo-spin*:

^{6}Li	$m_F=1/2$	maps to	spin 1/2
^{6}Li	$m_F=-1/2$	maps to	spin -1/2

- The up-and down spin particles *cannot* interchange — population imbalance is fixed. Possible spin states are:

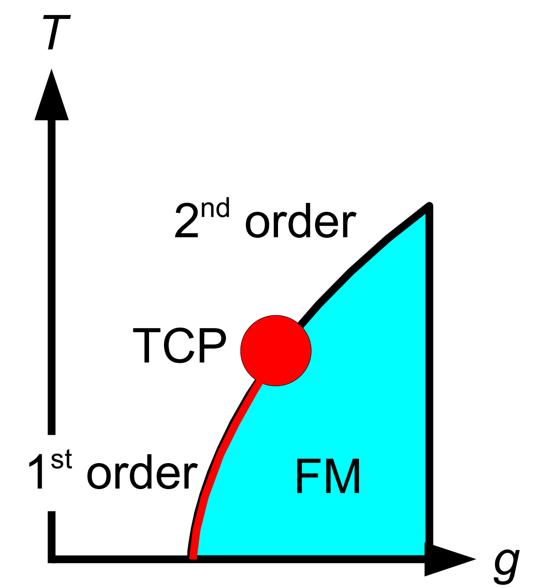
$\vert\uparrow\uparrow\rangle$	$S=1,\ S_z=1$	State not possible as S_z has changed
$\vert\downarrow\downarrow\rangle$	$S=1,\ S_z=-1$	State not possible as S_z has changed
$(\vert\uparrow\downarrow\rangle+\vert\downarrow\uparrow\rangle)/\sqrt{2}$	$S=1,\ S_z=0$	Magnetic moment in plane
$(\vert\uparrow\downarrow\rangle-\vert\downarrow\uparrow\rangle)/\sqrt{2}$	$S=0,\ S_z=0$	Non-magnetic state

- Ferromagnetism, if favourable, must form in-plane

Particle-hole perspective

- To second order in g the free energy is

$$F=\sum_{\sigma,\boldsymbol{k}}\epsilon_{\boldsymbol{k}}^{\sigma}n(\epsilon_{\boldsymbol{k}}^{\sigma})+gN^{\uparrow}N^{\downarrow}$$
$$-\frac{2g^2}{V^3}\sum_{\boldsymbol{p}}\int\int\frac{\rho^{\uparrow}(\boldsymbol{p},\epsilon_{\uparrow})\rho^{\downarrow}(-\boldsymbol{p},\epsilon_{\downarrow})}{\epsilon_{\uparrow}+\epsilon_{\downarrow}}d\epsilon_{\uparrow}d\epsilon_{\downarrow}$$
$$+\frac{2g^2}{V^3}\sum_{\boldsymbol{k}_{1,2,3,4}}\frac{n(\epsilon_{\boldsymbol{k}_1}^{\uparrow})n(\epsilon_{\boldsymbol{k}_2}^{\downarrow})}{\epsilon_{\boldsymbol{k}_1}^{\uparrow}+\epsilon_{\boldsymbol{k}_2}^{\downarrow}-\epsilon_{\boldsymbol{k}_3}^{\uparrow}-\epsilon_{\boldsymbol{k}_4}^{\downarrow}}\delta(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{k}_3-\boldsymbol{k}_3)$$



with $\epsilon_{\boldsymbol{k}}^{\sigma}=\epsilon_{\boldsymbol{k}}+\sigma gm$ and a particle-hole density of states

$$\rho^{\sigma}(\boldsymbol{p},\epsilon)=\sum_{\boldsymbol{k}}n(\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma})\left[1-n(\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma})\right]\delta\left(\epsilon-\epsilon_{\boldsymbol{k}+\boldsymbol{p}/2}^{\sigma}+\epsilon_{\boldsymbol{k}-\boldsymbol{p}/2}^{\sigma}\right)$$

- Enhanced particle-hole phase space at zero magnetisation drives transition first order
- Recover $m^4\ln m^2$ at T=0
- Links quantum fluctuation to second order perturbation approach[1]

[1]Abrikosov 1958 & Duine & MacDonald 2005

Quantum Monte Carlo: Textured phase

- Textured phase preempted transition with $q=0.2k_F$

***T*=0**

Modified collective modes

- Collective mode dispersion

$$\Omega = \frac{q^2}{2}\left(1 - \frac{2^{5/3}3}{5k_{\mathrm{F}}a}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}\right)$$

- Collective mode damping

$$\Gamma = \frac{q^2}{2}\frac{2^{5/3}3\tilde{\lambda}}{5(k_{\mathrm{F}}a)^2}\frac{1}{1+\tilde{\lambda}^2/(k_{\mathrm{F}}a)^2}$$