UNIVERSITY OF CAMBRIDGE

Few-atom approach to many-body physics

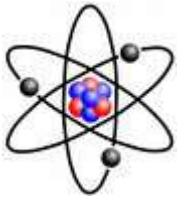
G.J. Conduit and P.O. Bugnion

POB, J. Lofthouse & GJC, Phys. Rev. Lett. **111**, 045301 (2013)

POB & GJC, Phys. Rev. A **87**, 060502(R) (2013)

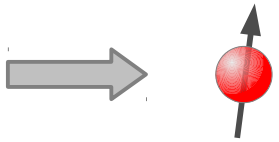
POB & GJC, Phys. Rev. A **88**, 013601 (2013)

Experimental setup



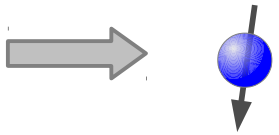
^{6}Li atom

$$|F = 1/2, m_F = 1/2\rangle$$



Up spin electron

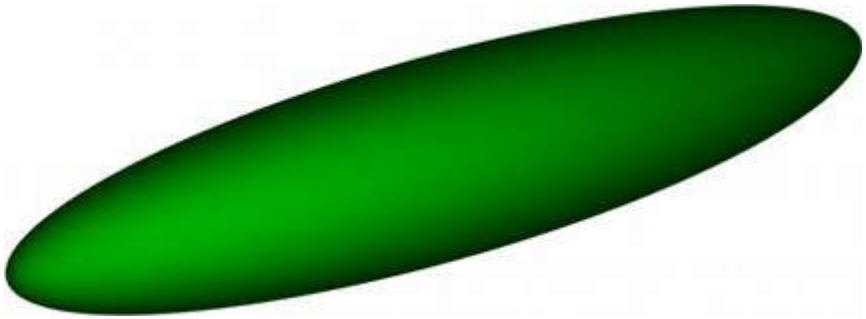
$$|F = 1/2, m_F = -1/2\rangle$$



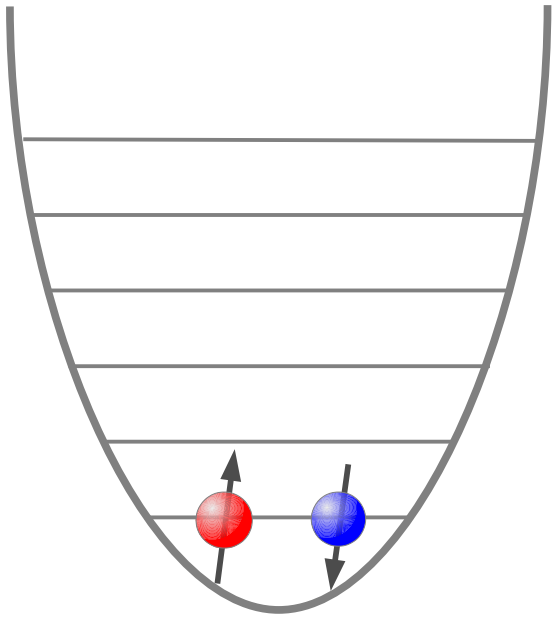
Down spin electron

$$\hat{H} = -\frac{\nabla^2}{2} + g n_{\uparrow}(\vec{r}) n_{\downarrow}(\vec{r})$$

Experimental setup

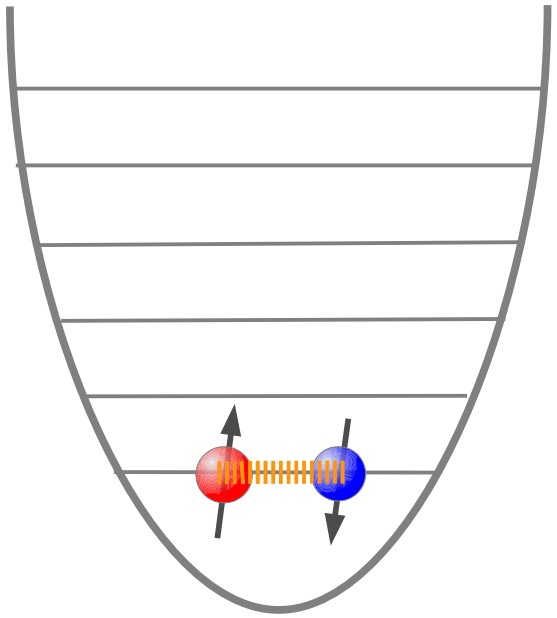


Two distinguishable fermions

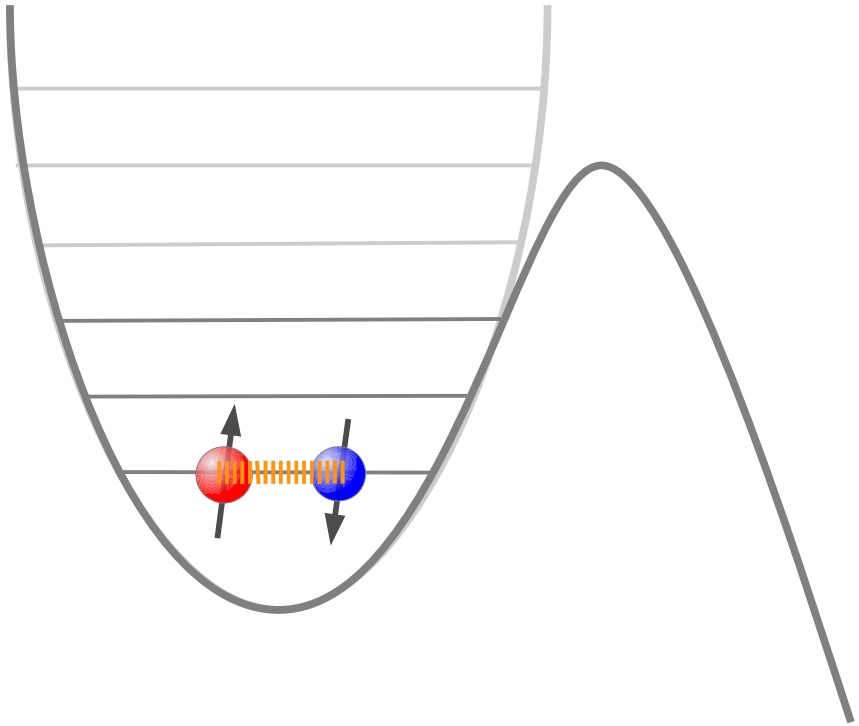


G. Zürn *et al*. PRL 108 075303 (2012)

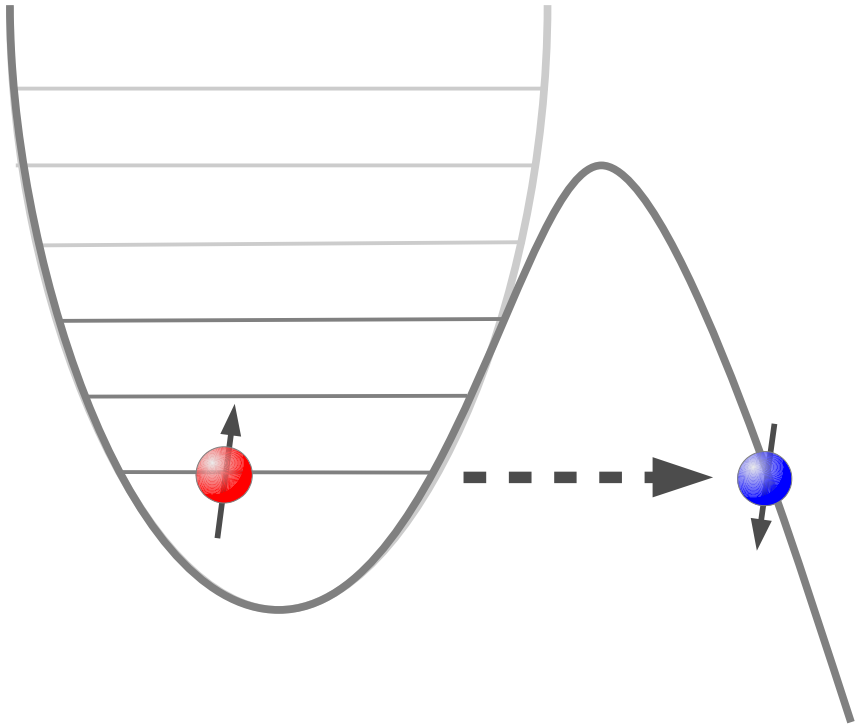
Two distinguishable fermions



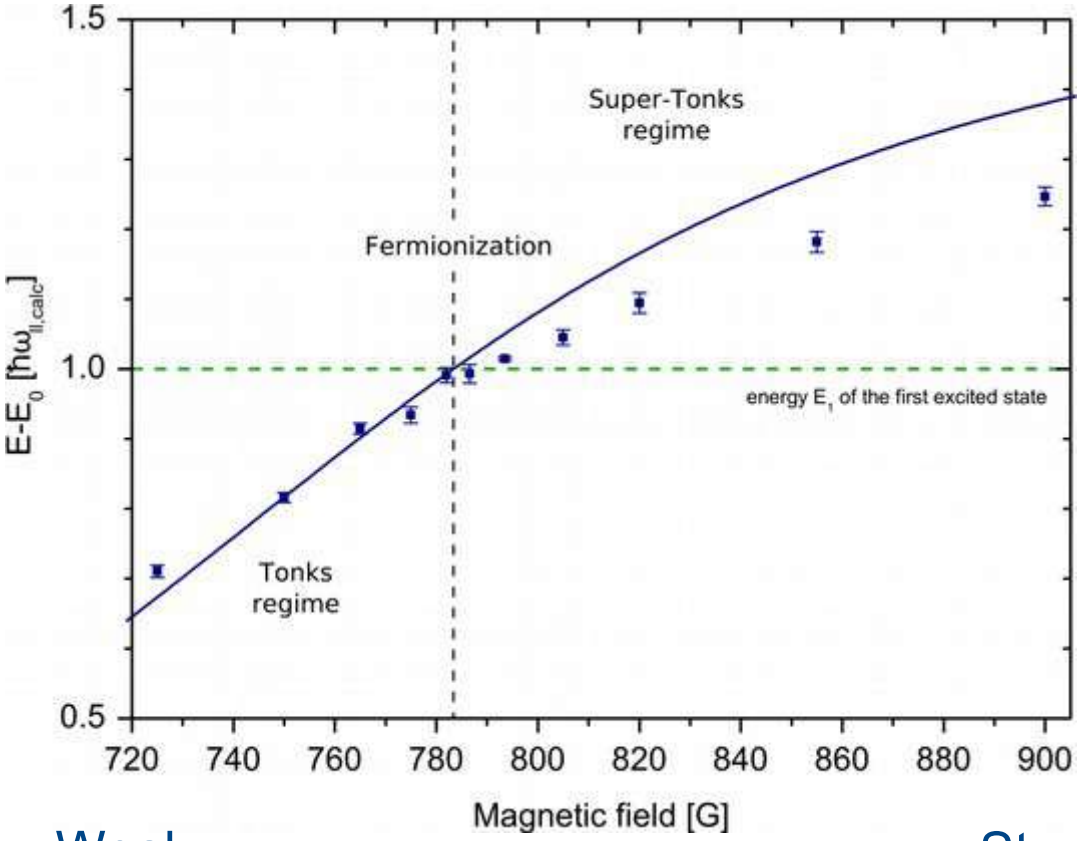
Two distinguishable fermions

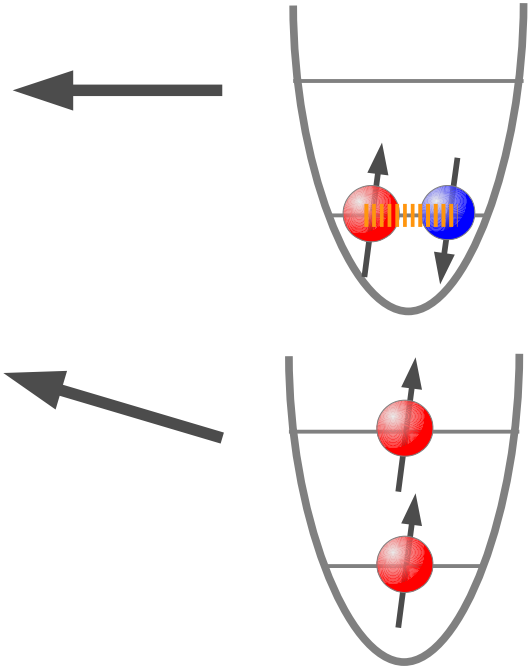


Two distinguishable fermions



Energy of states

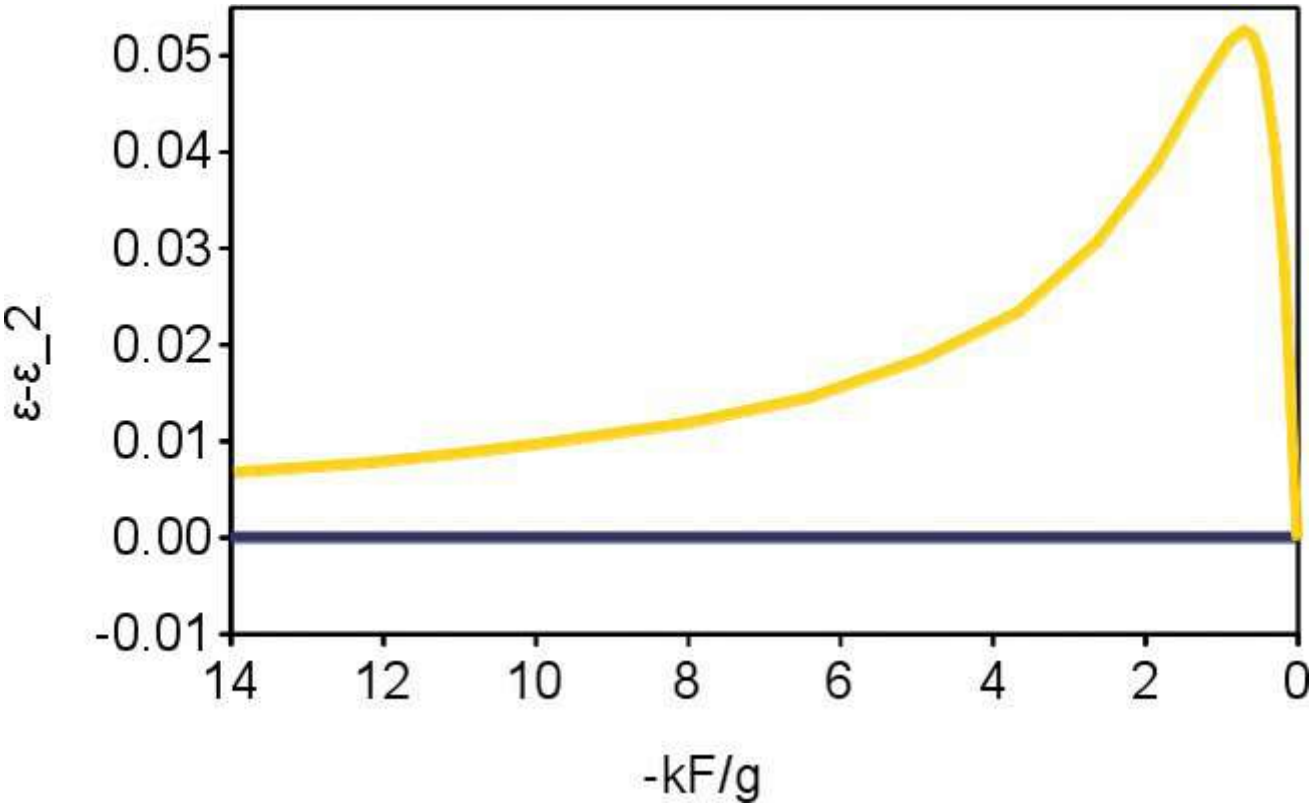


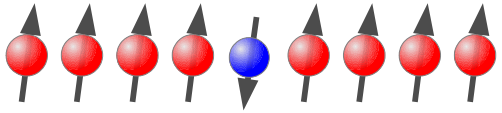


Weak repulsion

Strong repulsion

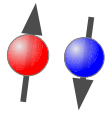
Polaron state





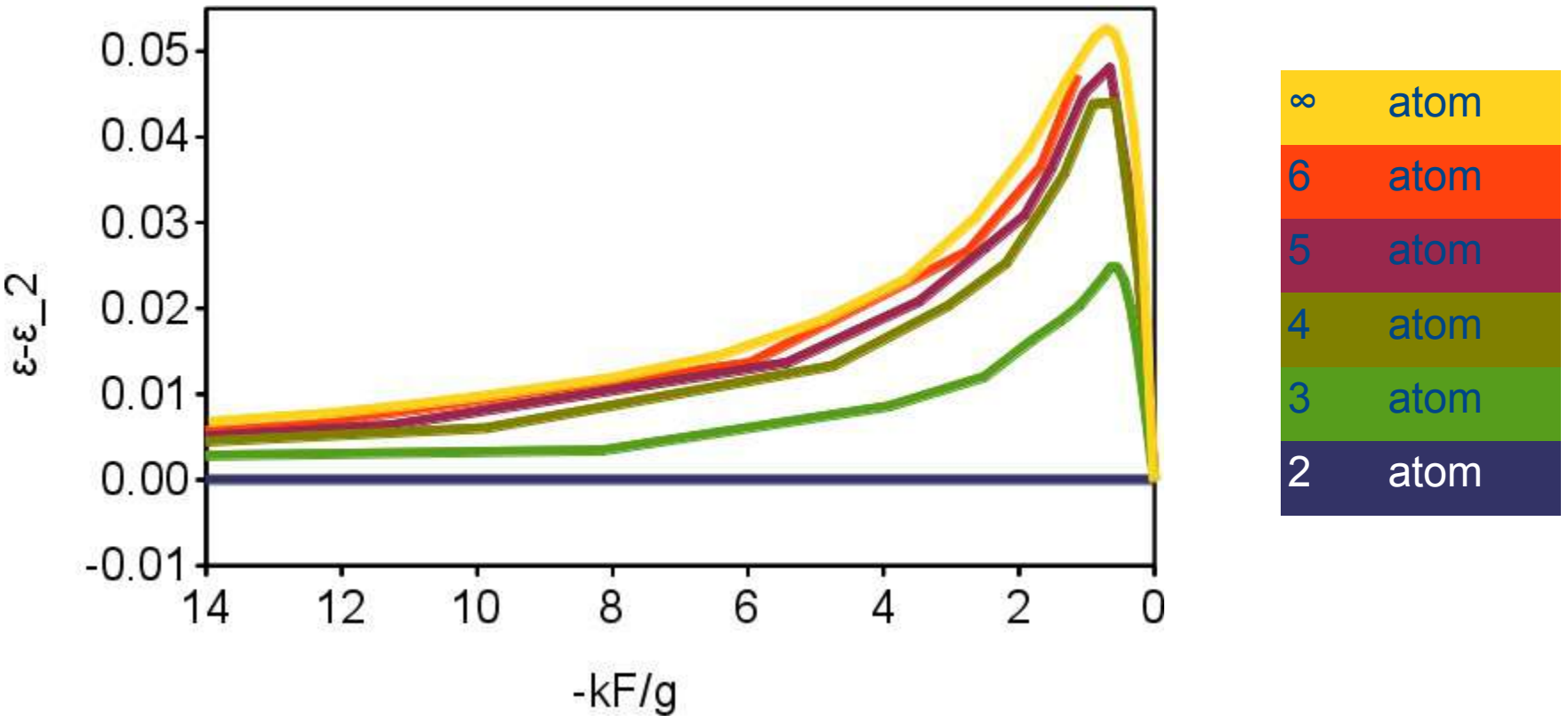
∞ atom

2 atom



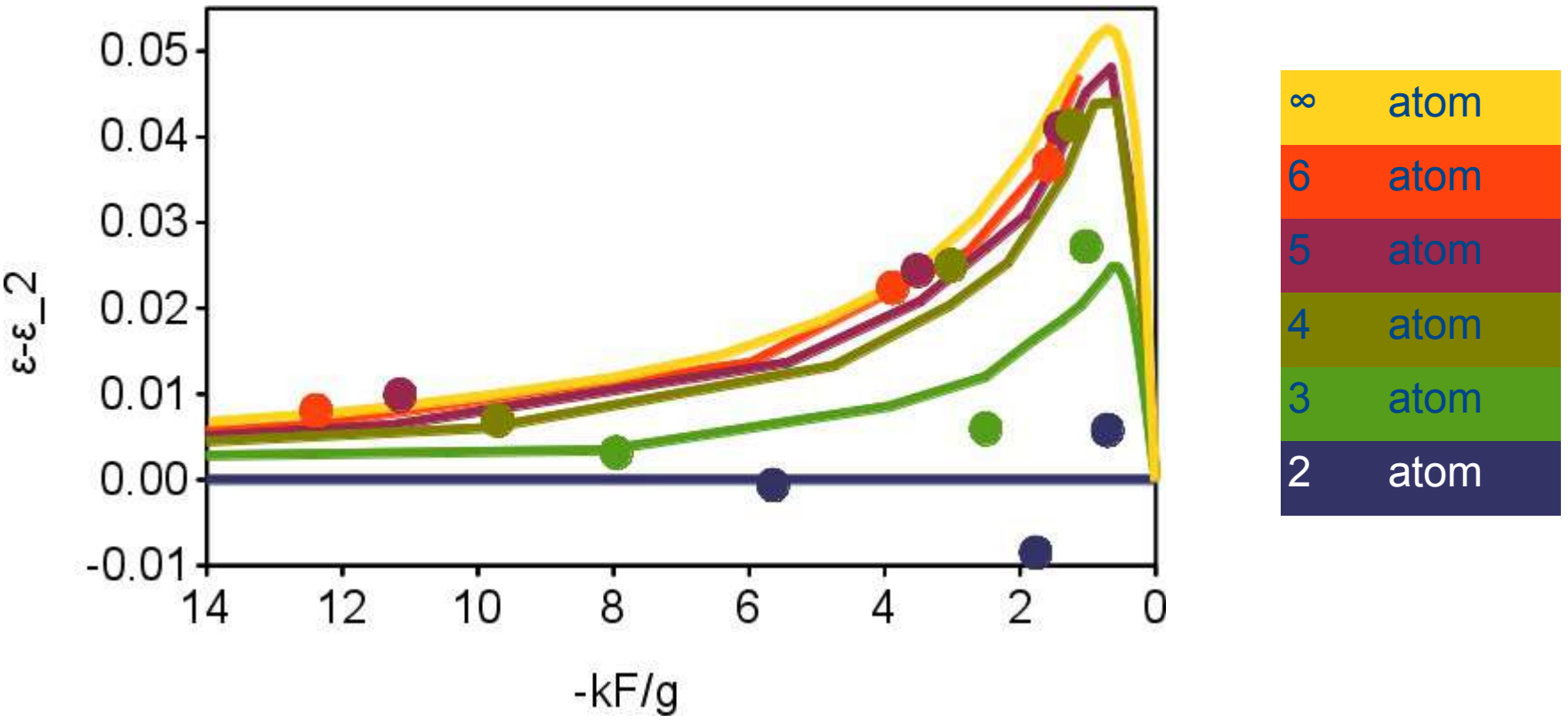
J.B. McGuire, J. Math. Phys. **6**, 432 (1965)

Polaron state



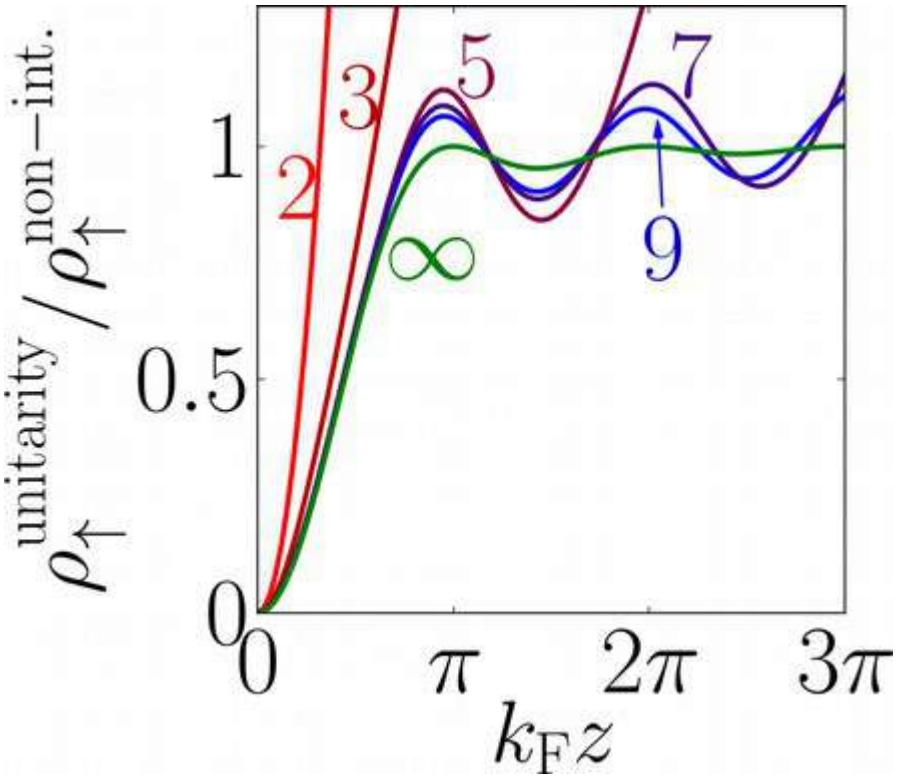
P.O. Bugnion & G.J. Conduit Phys. Rev. A **87**, 060502(R) (2013)

Polaron state



A. Wenz *et al.*, arXiv:1307.3443

Polaron state



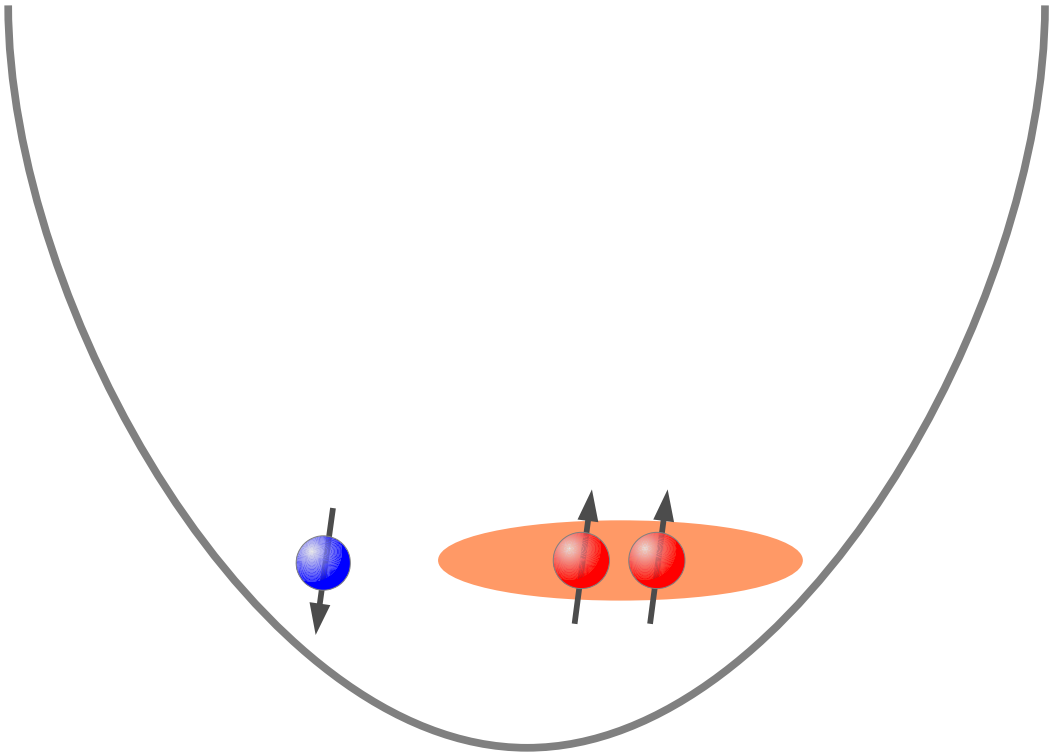
Strongly correlated physics

Repulsive interactions	→	Ferromagnetic correlations
Attractive interactions	→	BCS pairing / FFLO
Double well	→	Exchange interactions
Zero dimensional	→	Hund's rules
Perturbed trap	→	Rabi / Josephson oscillations

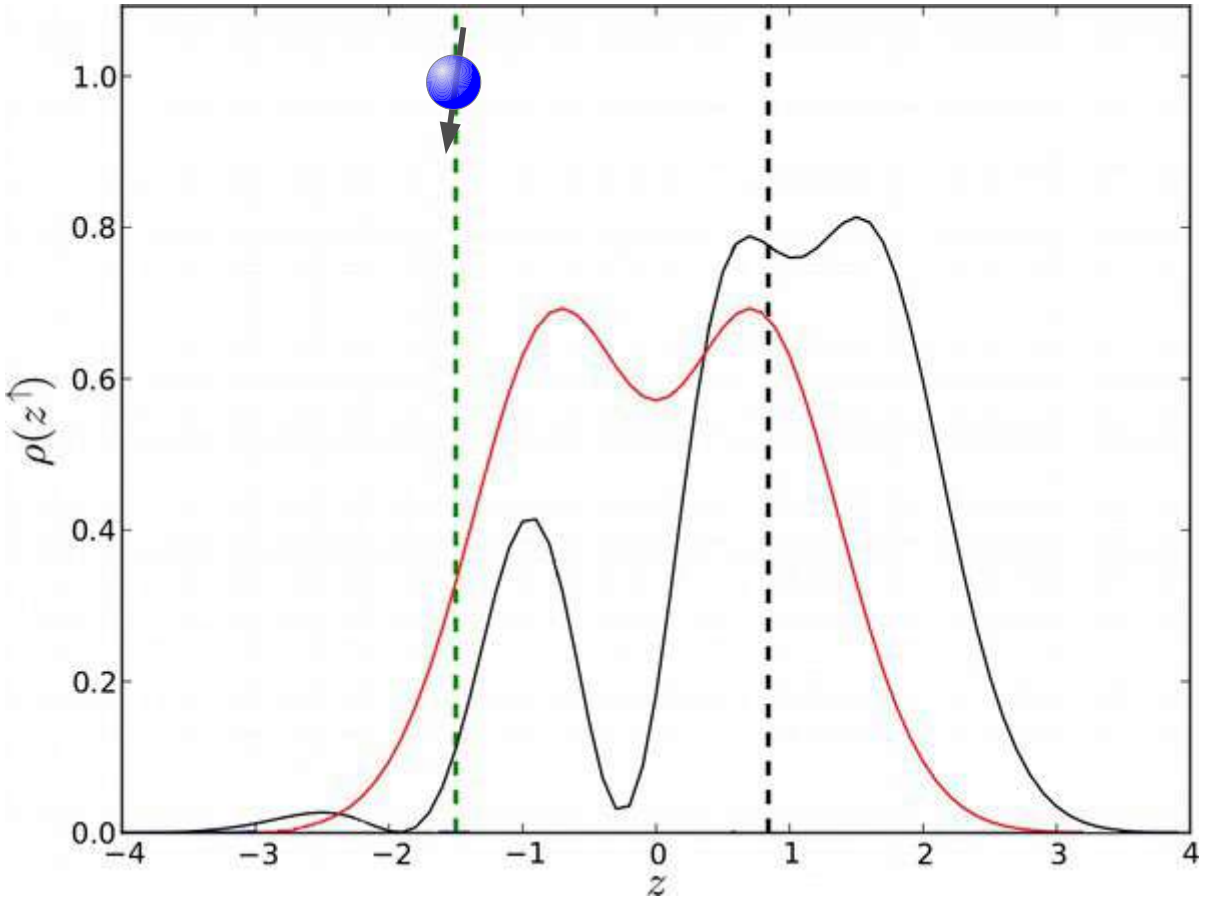
Stoner Hamiltonian

$$\hat{H} = -\frac{\nabla^2}{2} + g n_{\uparrow}(\vec{r})\, n_{\downarrow}(\vec{r}) + V(\vec{r})$$

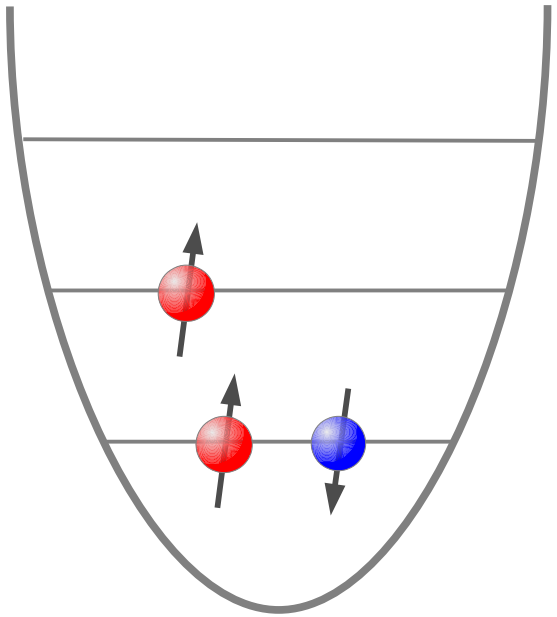
Density profiles



Density profiles



Polaron state



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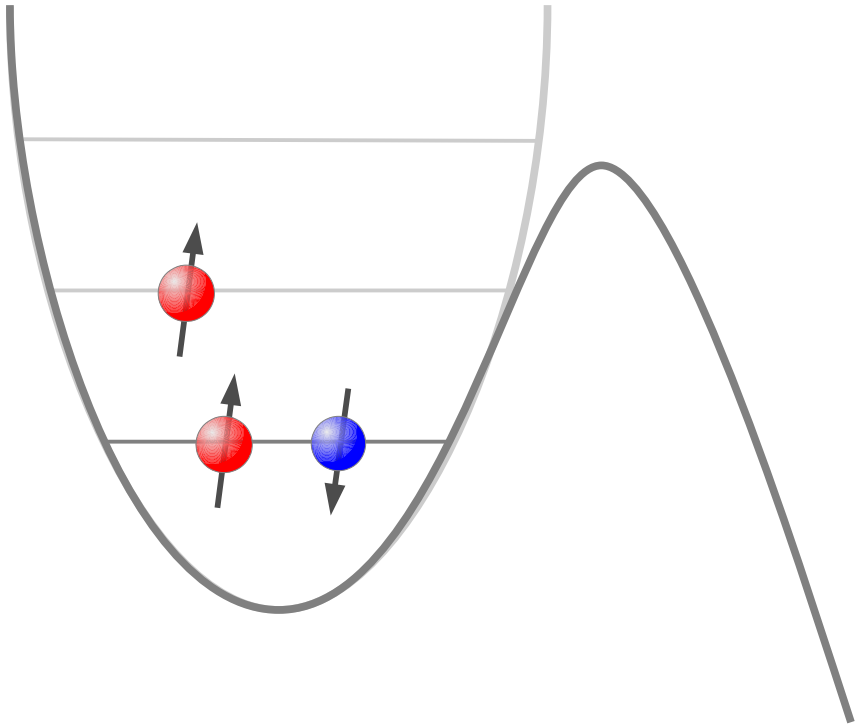
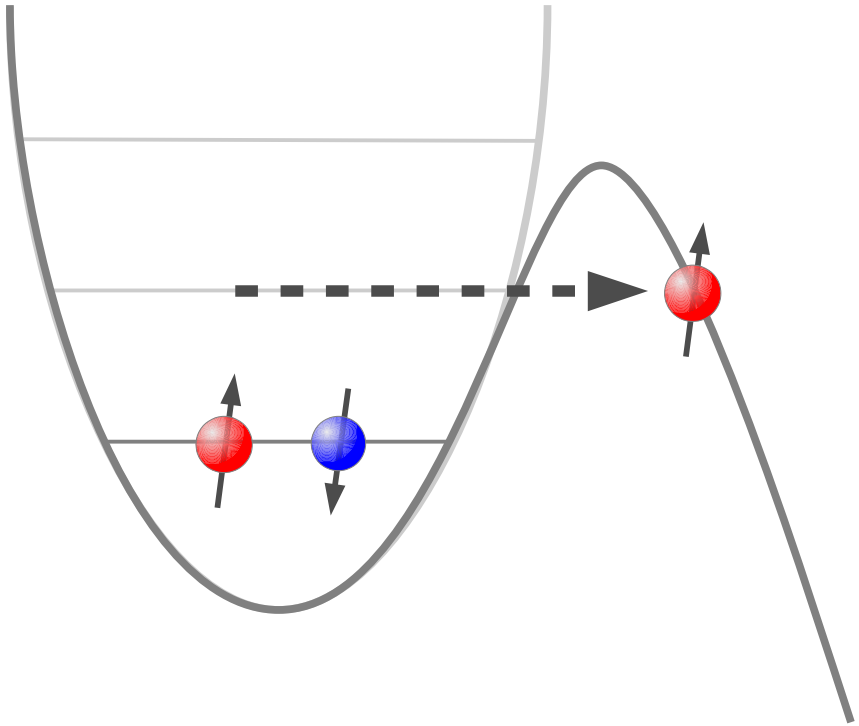
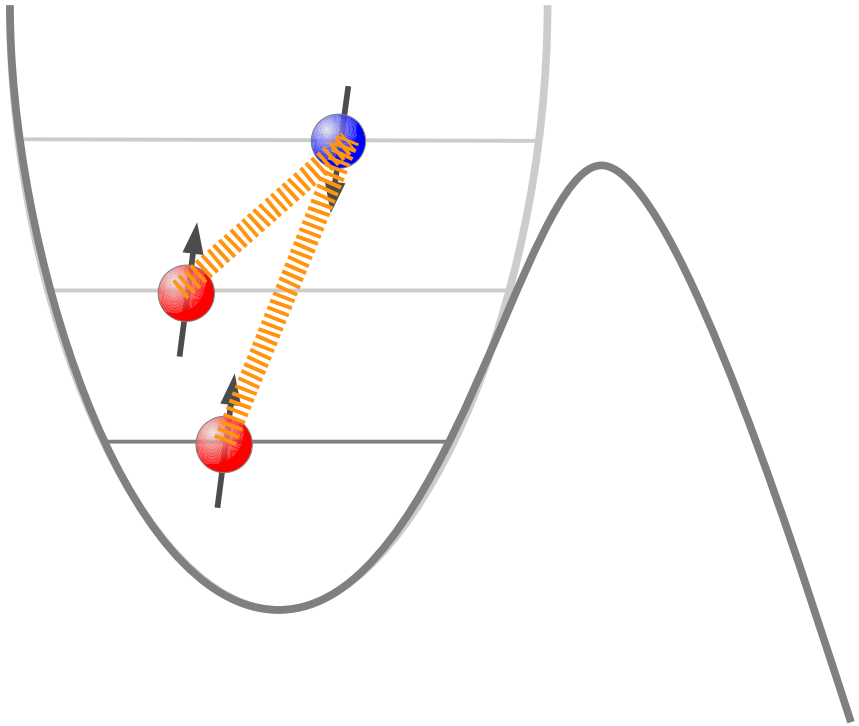


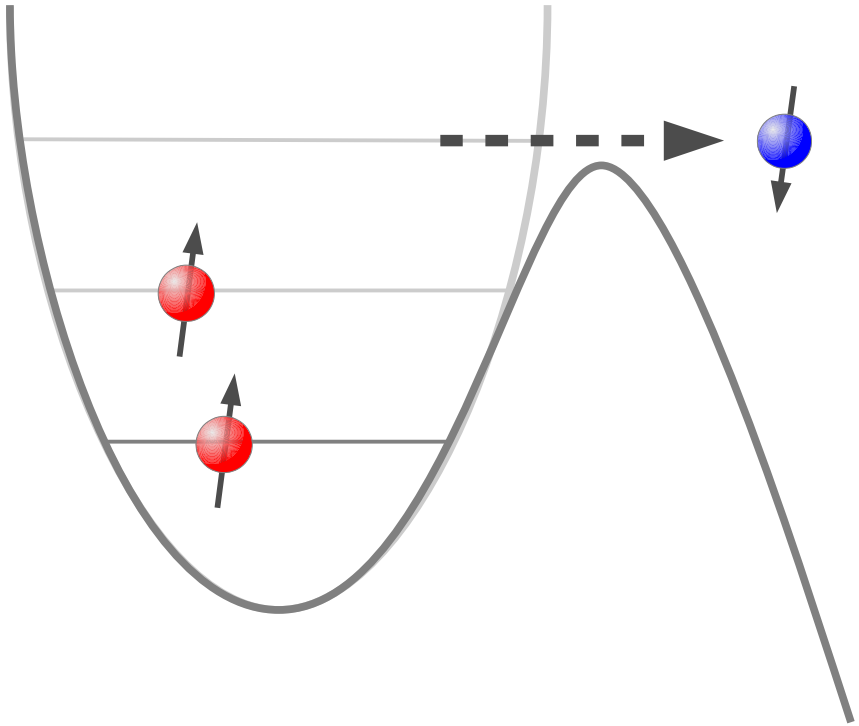
Polaron state



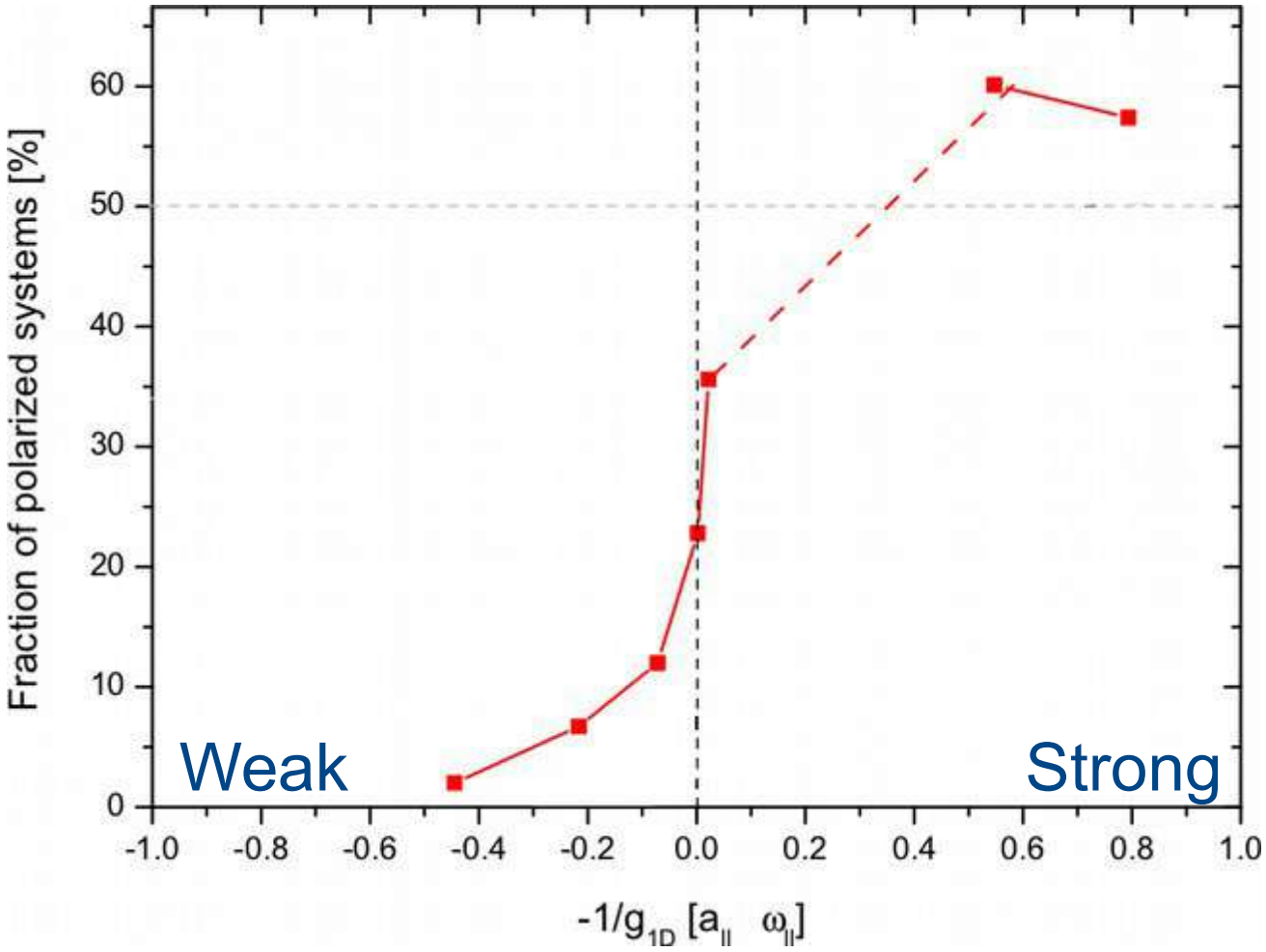
Polaron state

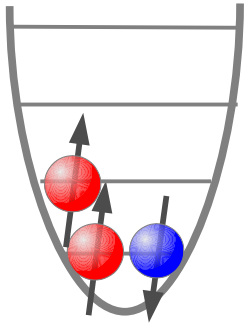


Polaron state

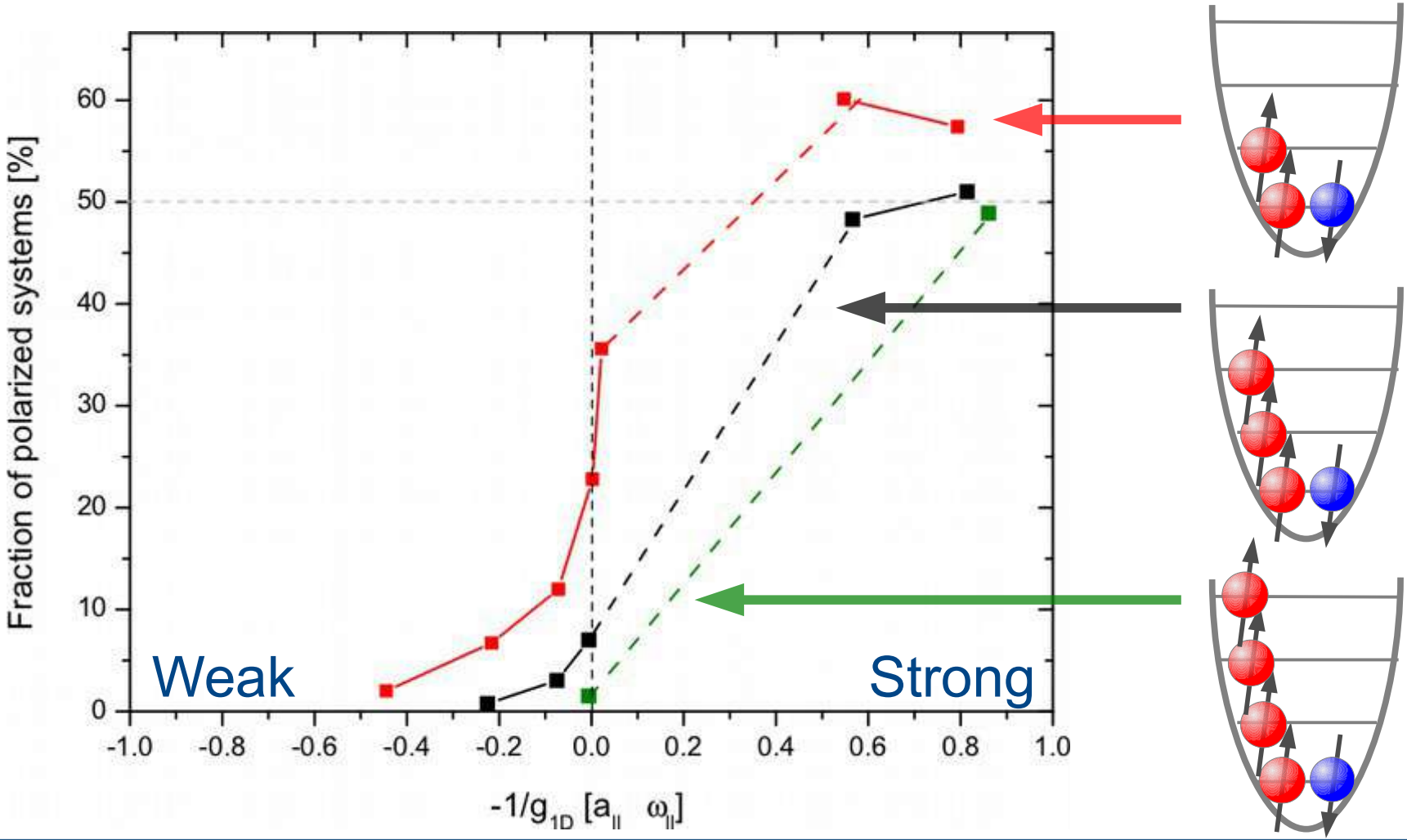


Loss region

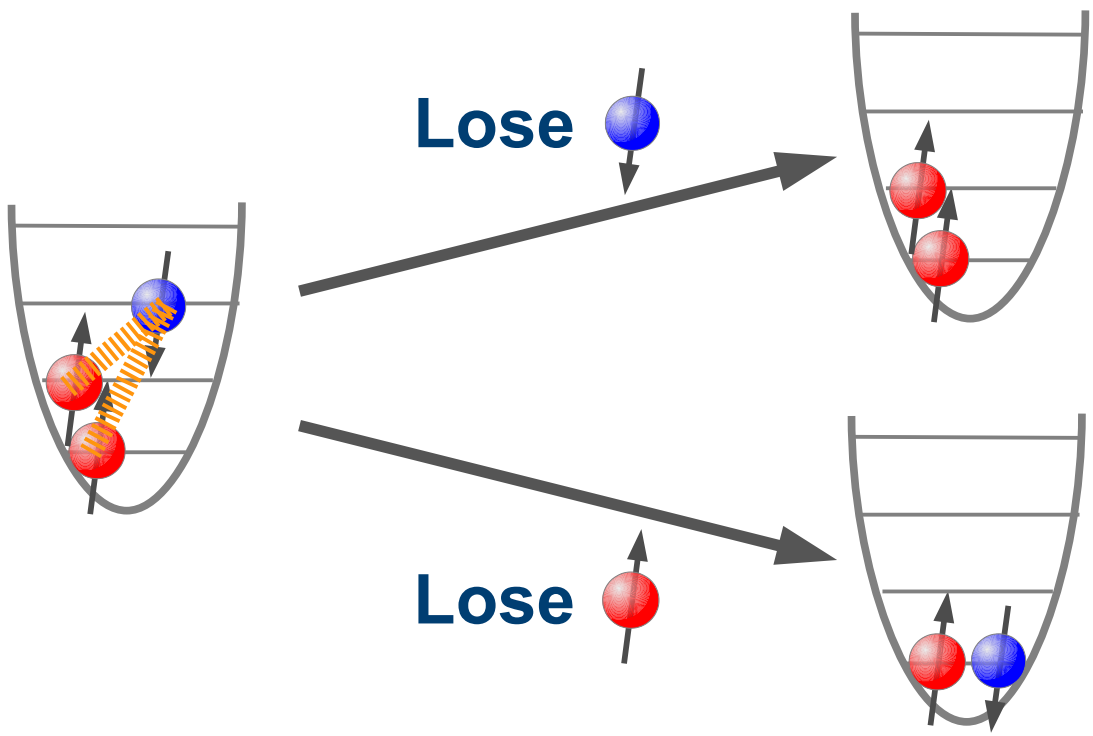

Fraction of polarized systems [%]
60
50
40
30
20
10
0
-1.0
-0.8
-0.6
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
Weak
Strong
-1/g1D [a⊥ ω⊥]



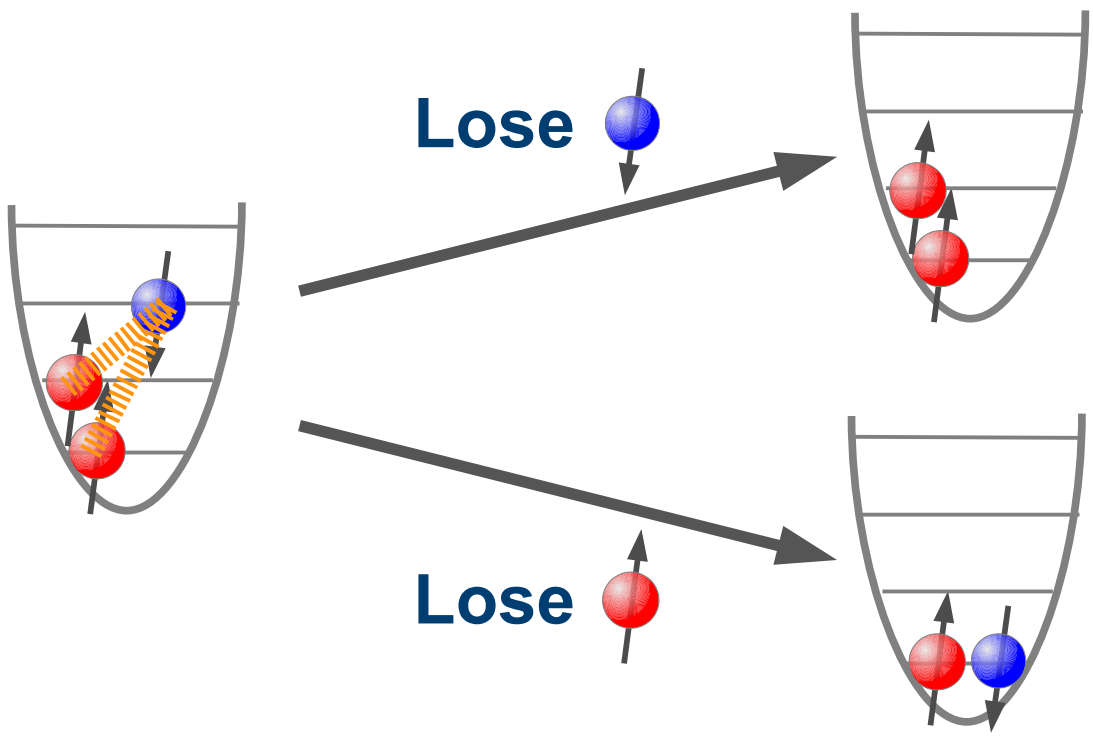
Tunneling probability



Why probability of ½?



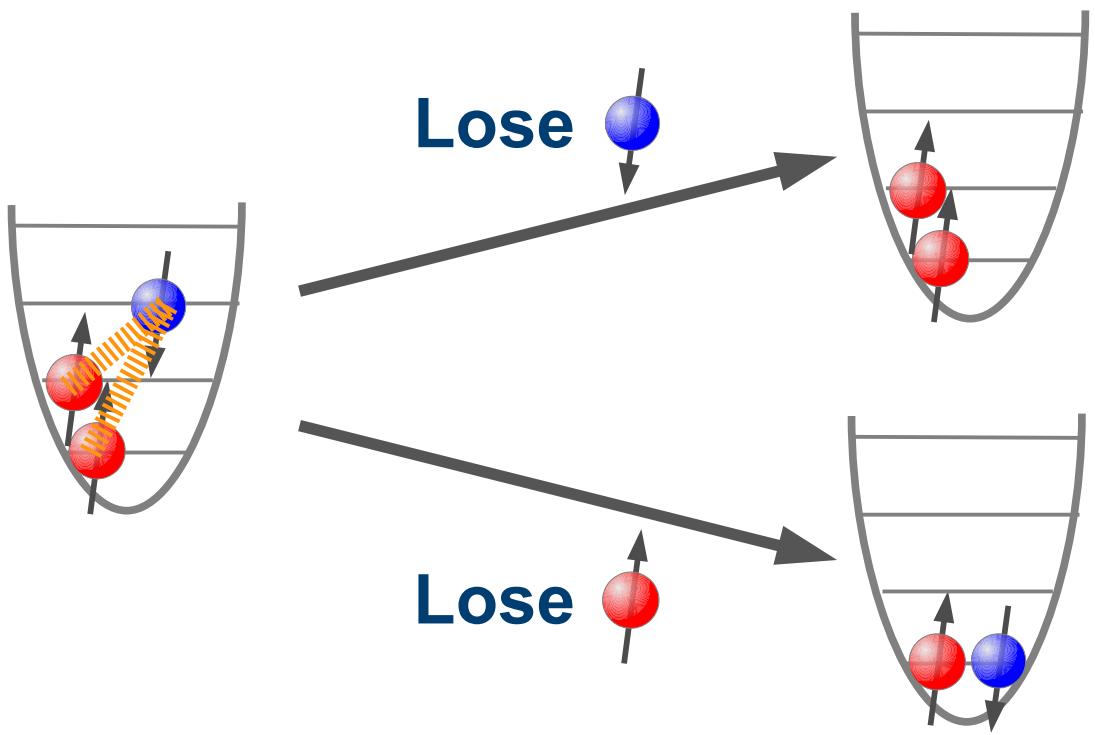
Why probability of ½?



$|\uparrow\uparrow\rangle$

Ferro

Why probability of ½?

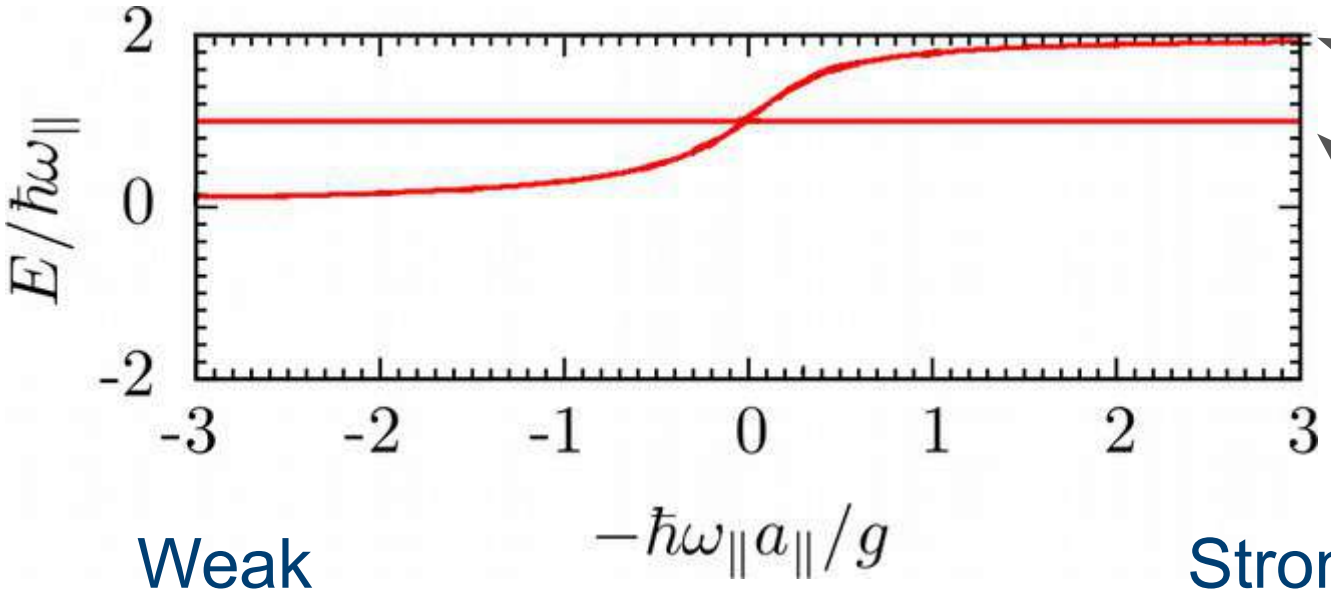


$|\uparrow\uparrow\rangle$ Ferro

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ Ferro

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ Para

Finding the missing probability



Weak

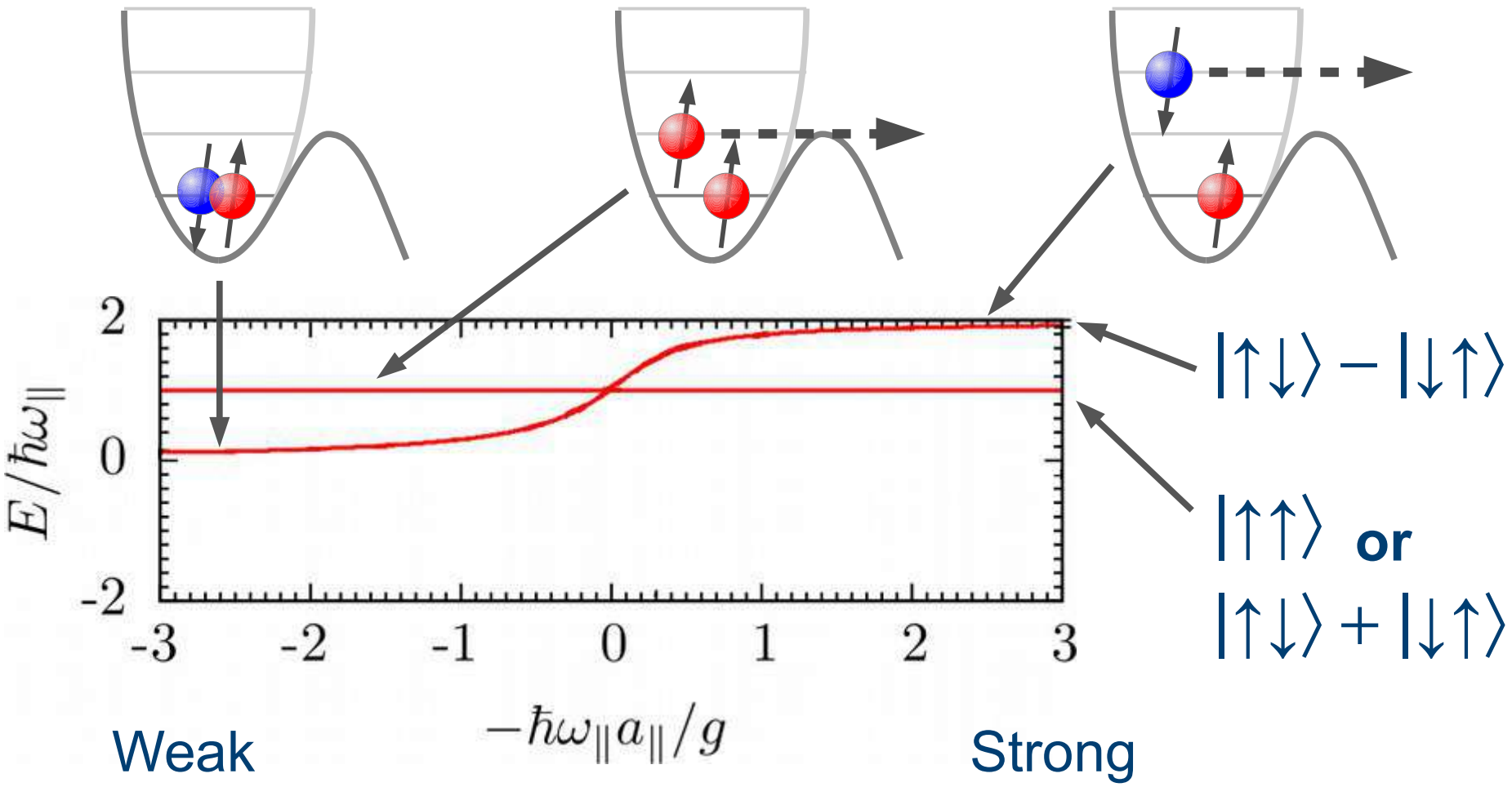
Strong

$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$

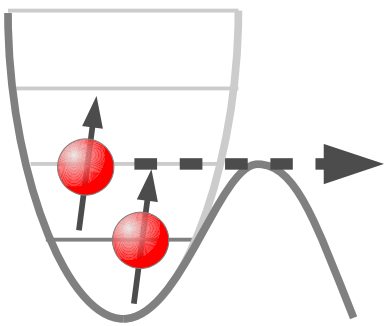
$|\uparrow\uparrow\rangle$ **or**

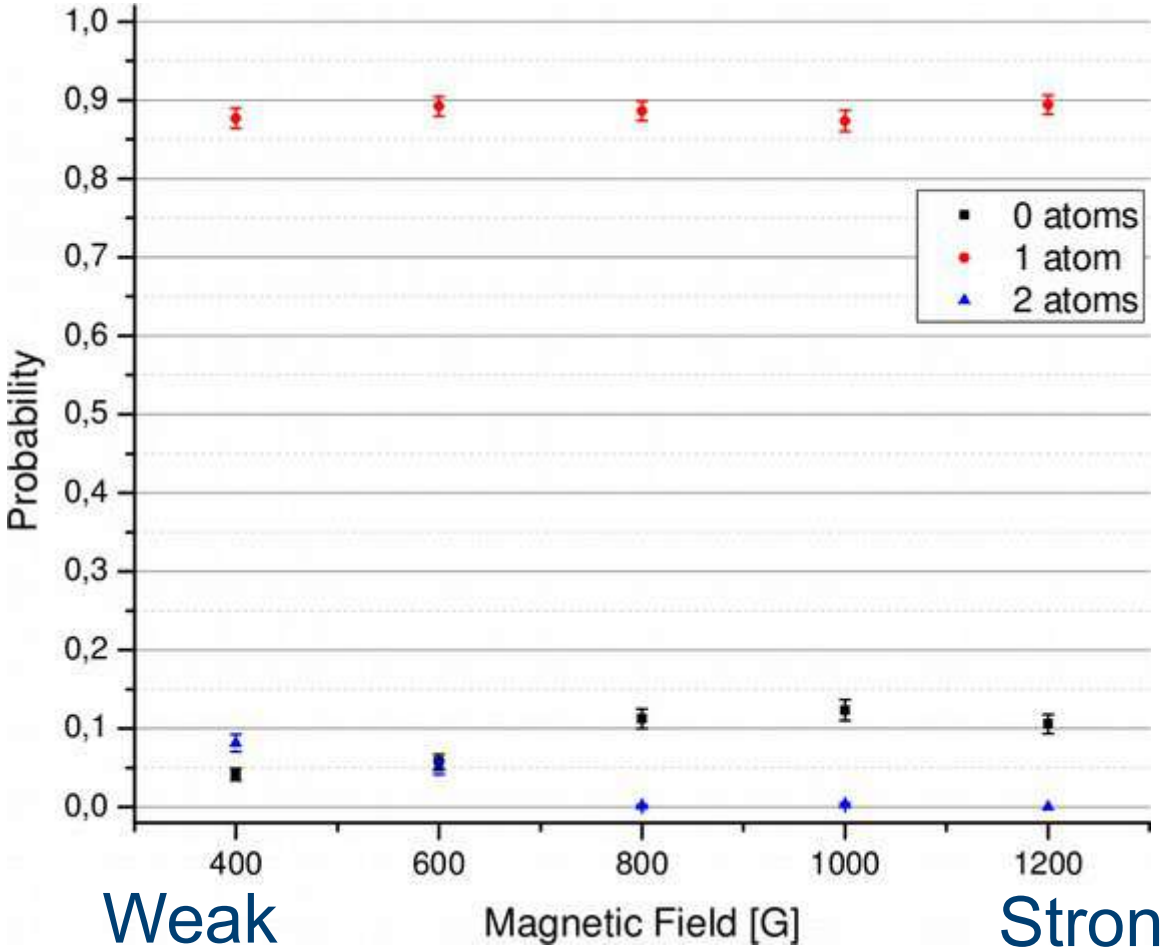
$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$

Finding the missing probability



Tunneling probability

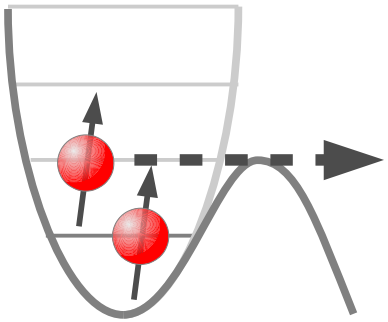




Weak

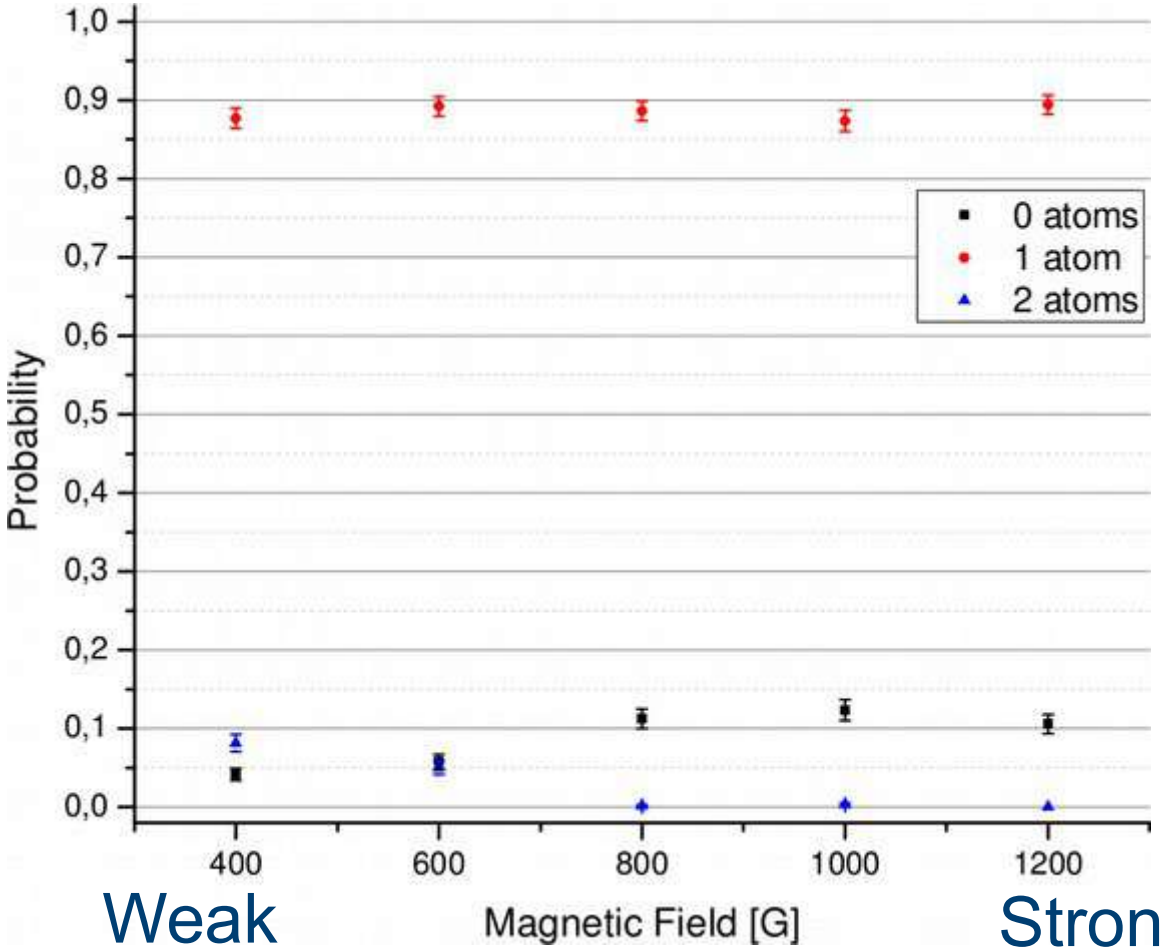
Strong

Tunneling probability



$|\uparrow\uparrow\rangle$ **or**

$|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$



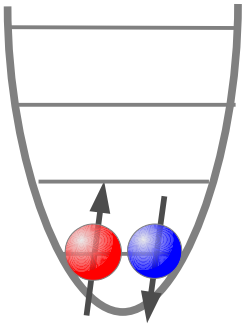
Lieb-Mattis theorem

PHYSICAL REVIEW VOLUME 125, NUMBER 1 JANUARY 1, 1962

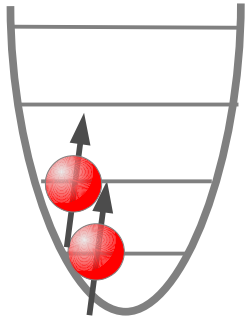
Theory of Ferromagnetism and the Ordering of Electronic Energy Levels

ELLIOTT LIEB AND DANIEL MATTIS
Thomas J. Watson Research Center, International Business Machines Corporation, Yorktown Heights, New York
(Received May 25, 1961; revised manuscript received September 11, 1961)

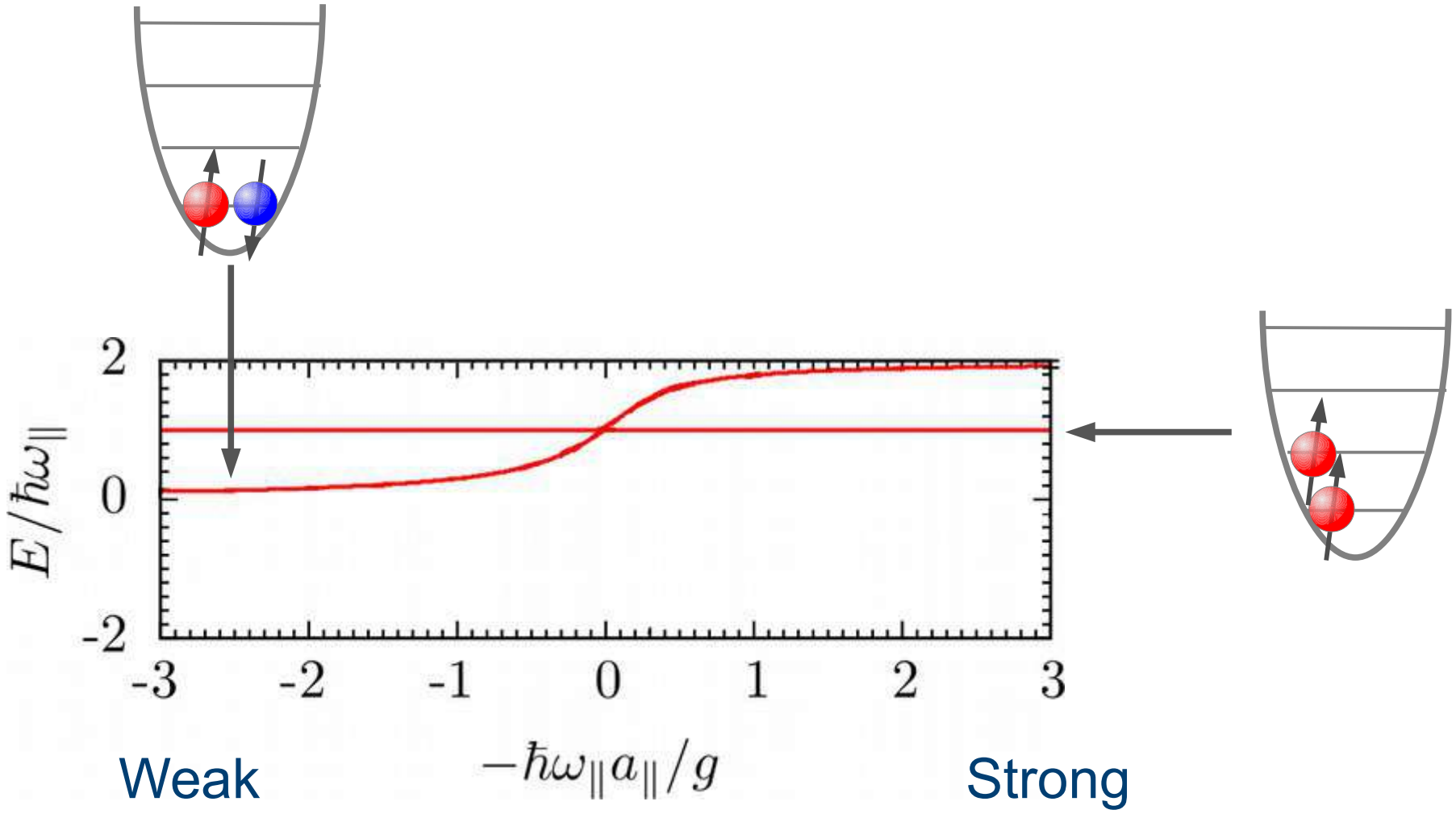
Consider a system of N electrons in one dimension subject to an arbitrary symmetric potential, $V(x_1,\cdots,x_N)$, and let $E(S)$ be the lowest energy belonging to the total spin value S. We have proved the following theorem: $E(S)<E(S')$ if $S<S'$. Hence, the ground state is unmagnetized. The theorem also holds



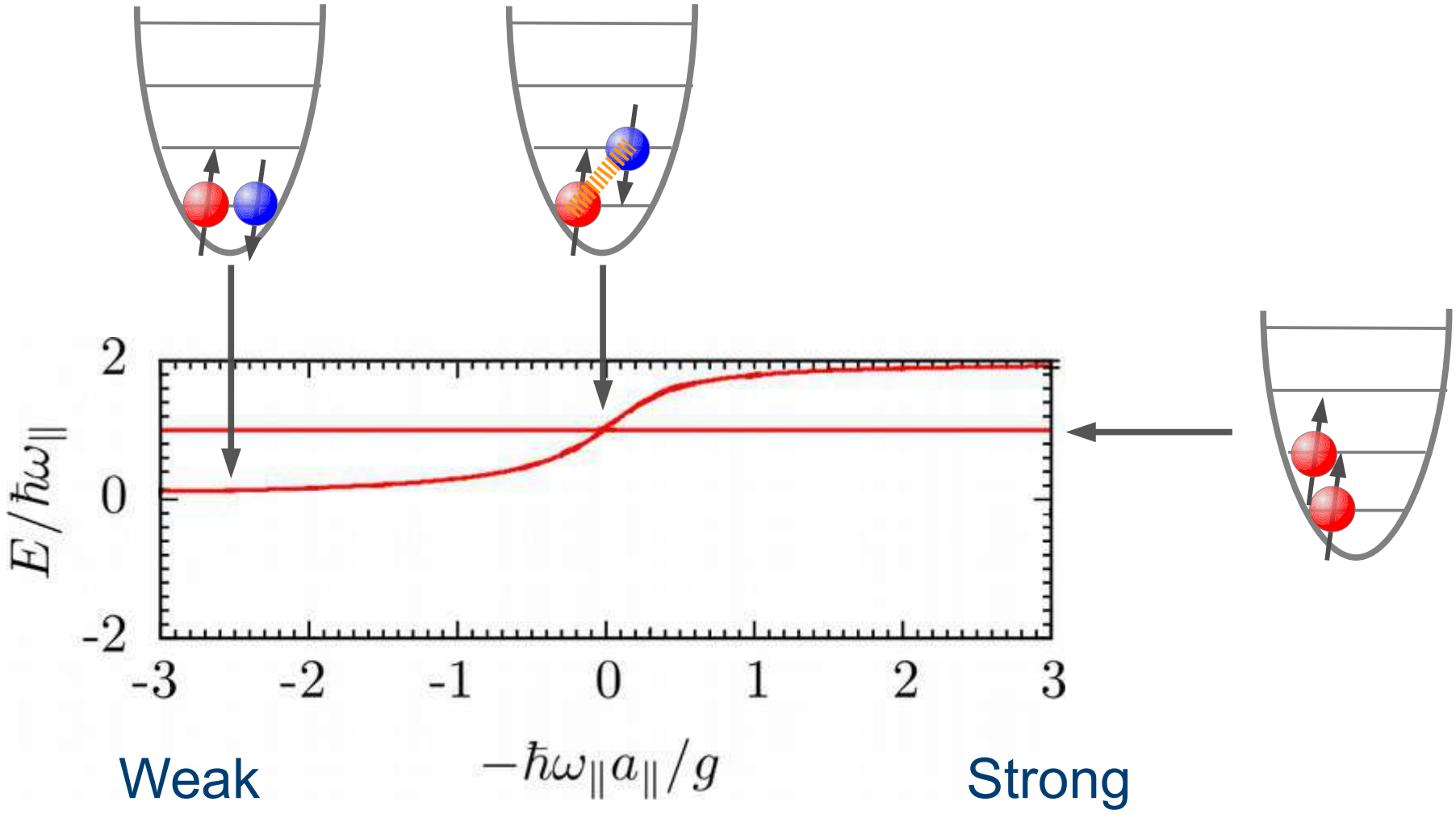
has lower energy than



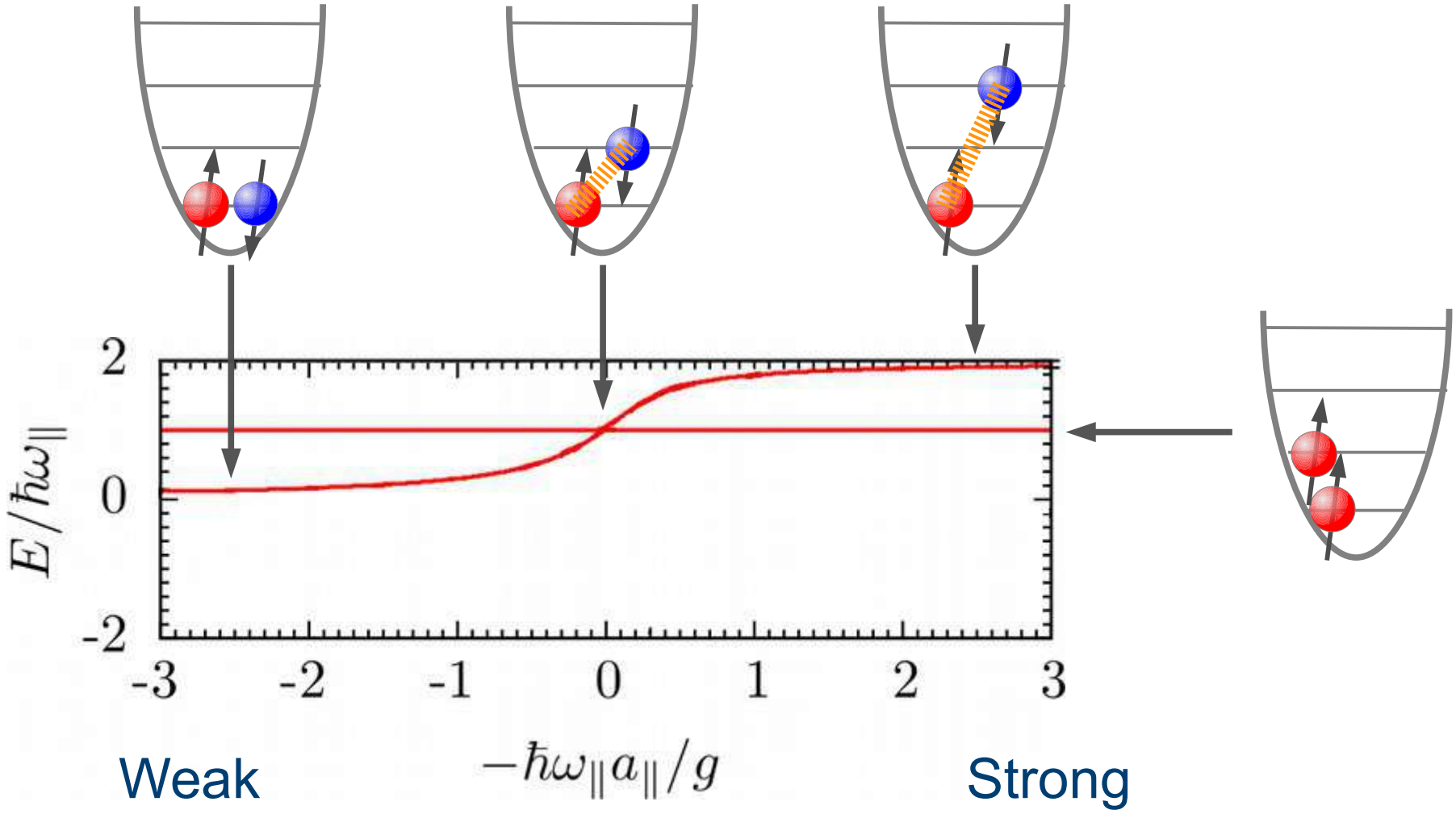
Lieb-Mattis theorem



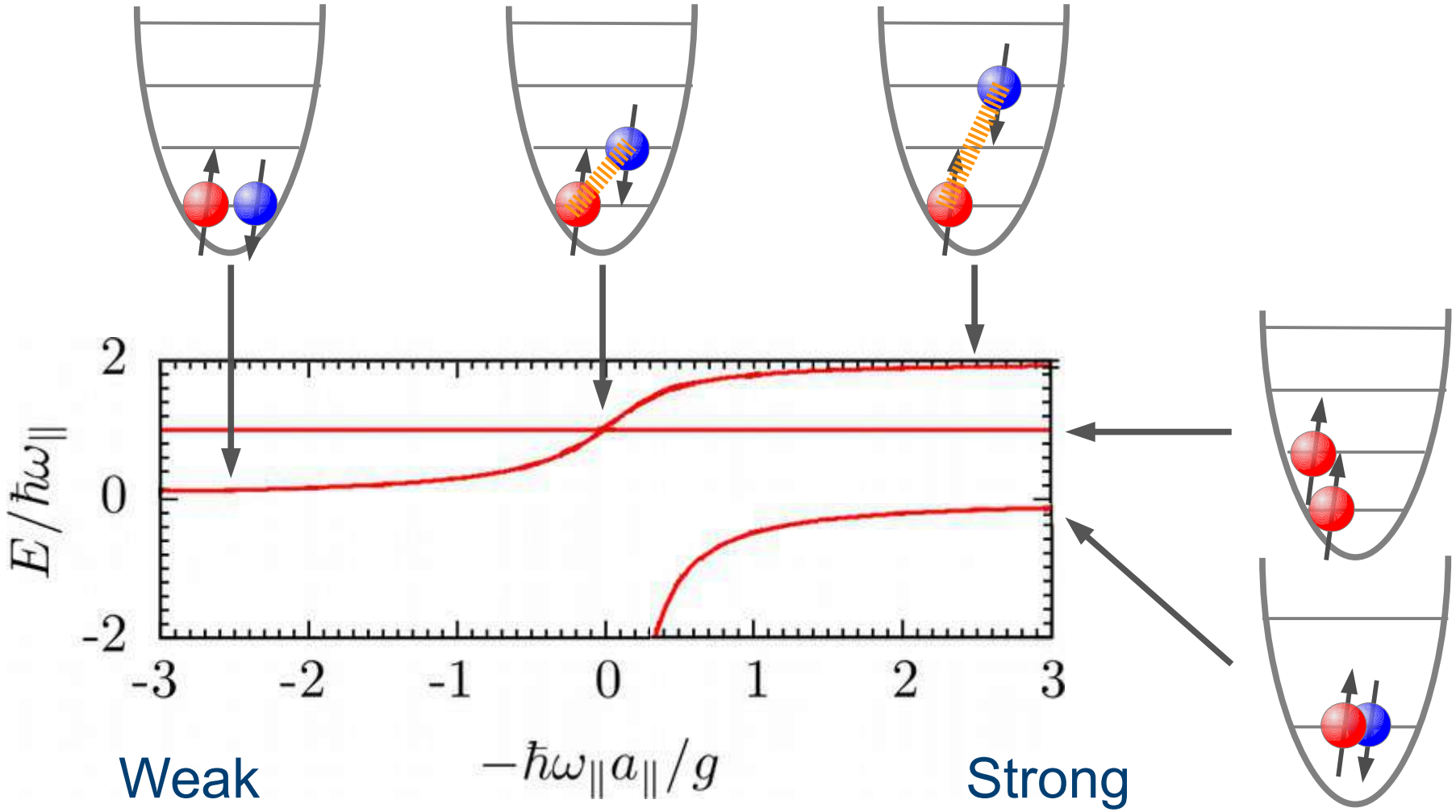
Lieb-Mattis theorem



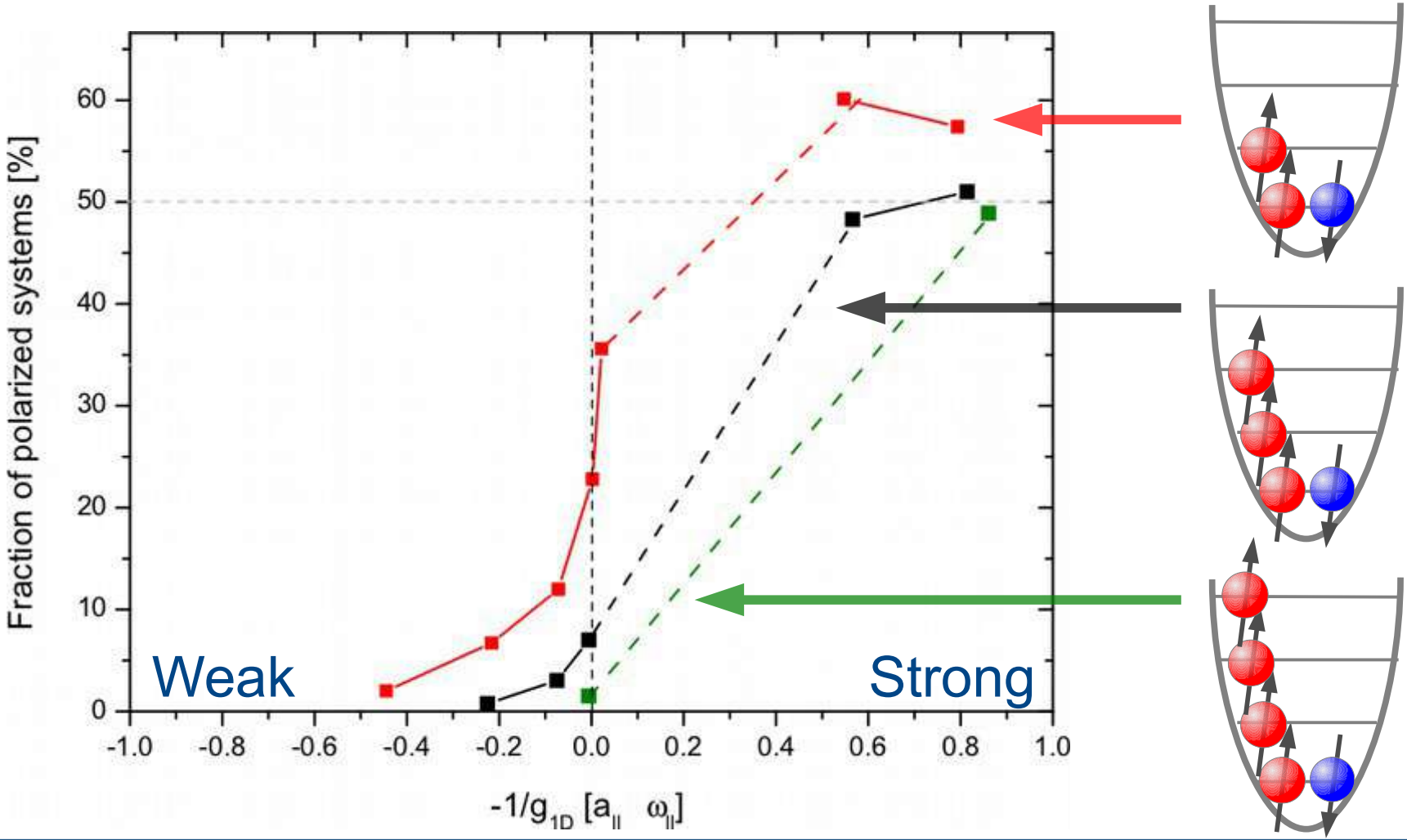
Lieb-Mattis theorem



Lieb-Mattis theorem

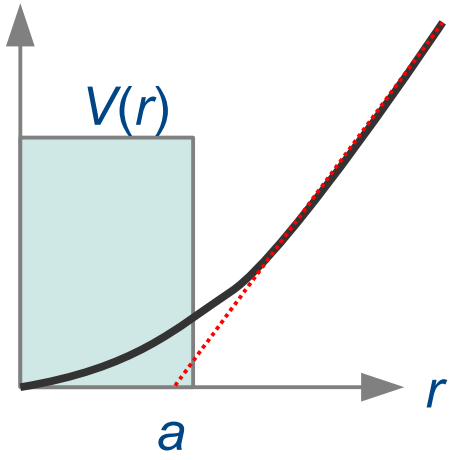


Tunneling probability



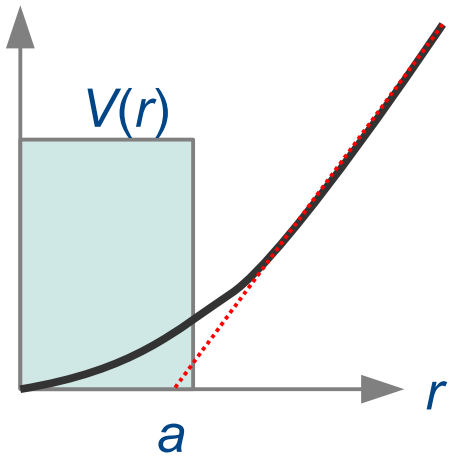
Two-atom scattering

Repulsive

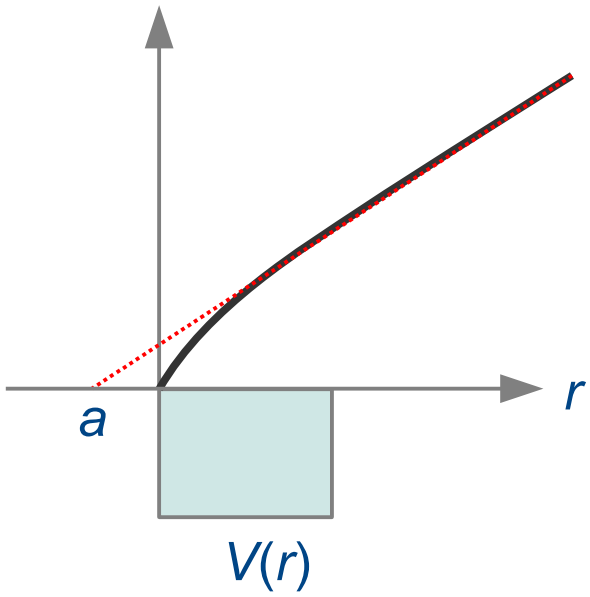


Two-atom scattering

Repulsive

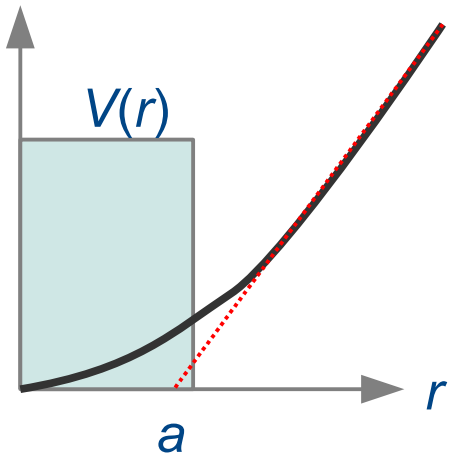


Attractive



Two-atom scattering

Repulsive



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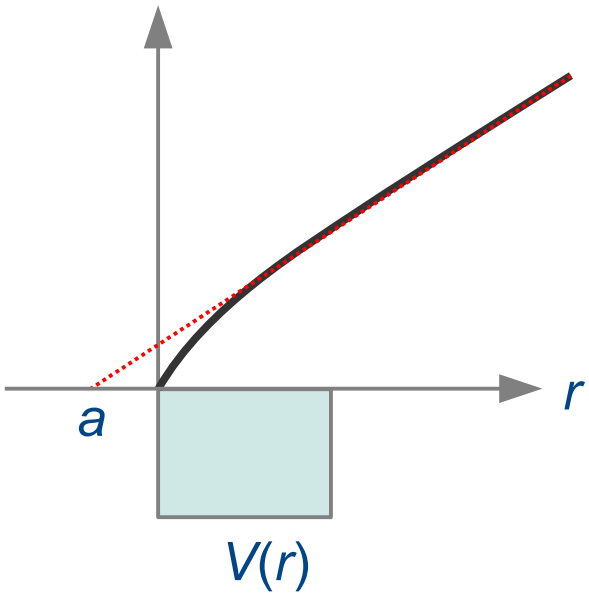
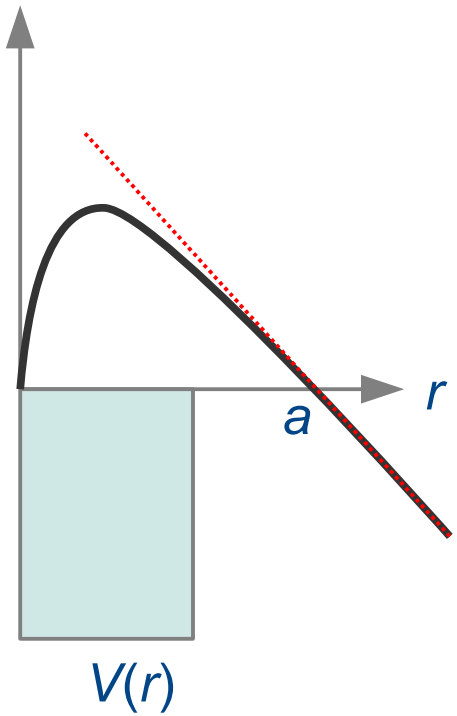
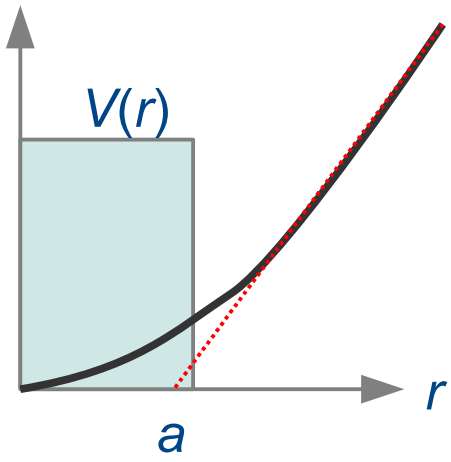


Repulsive

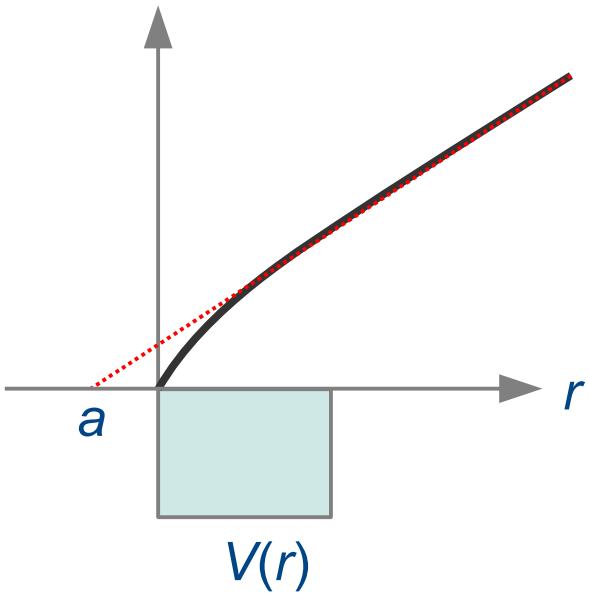


Two-atom scattering

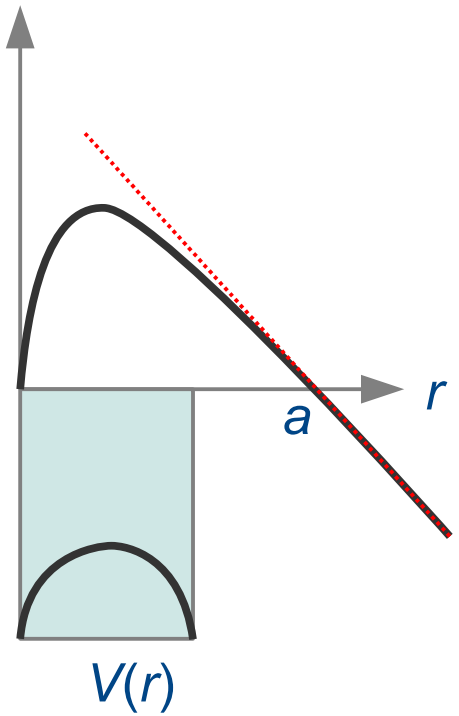
Repulsive



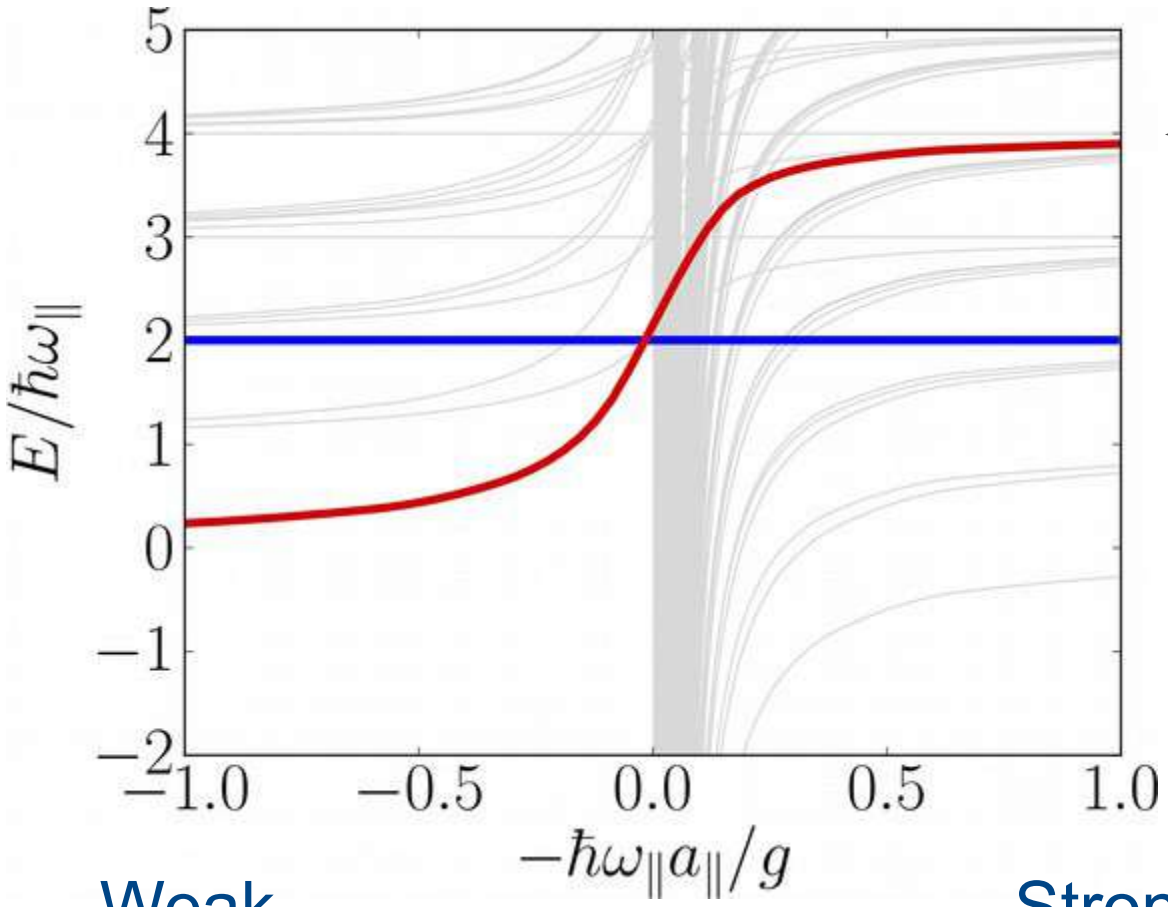
Attractive

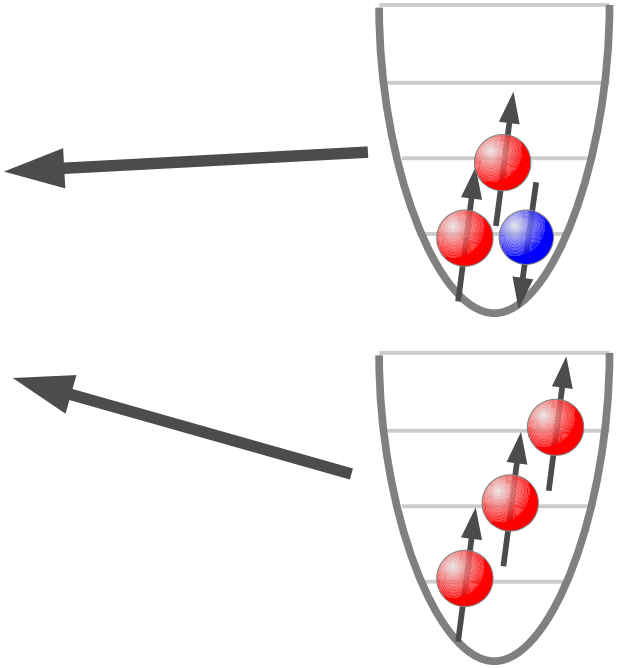


Repulsive



Three-atom bound state

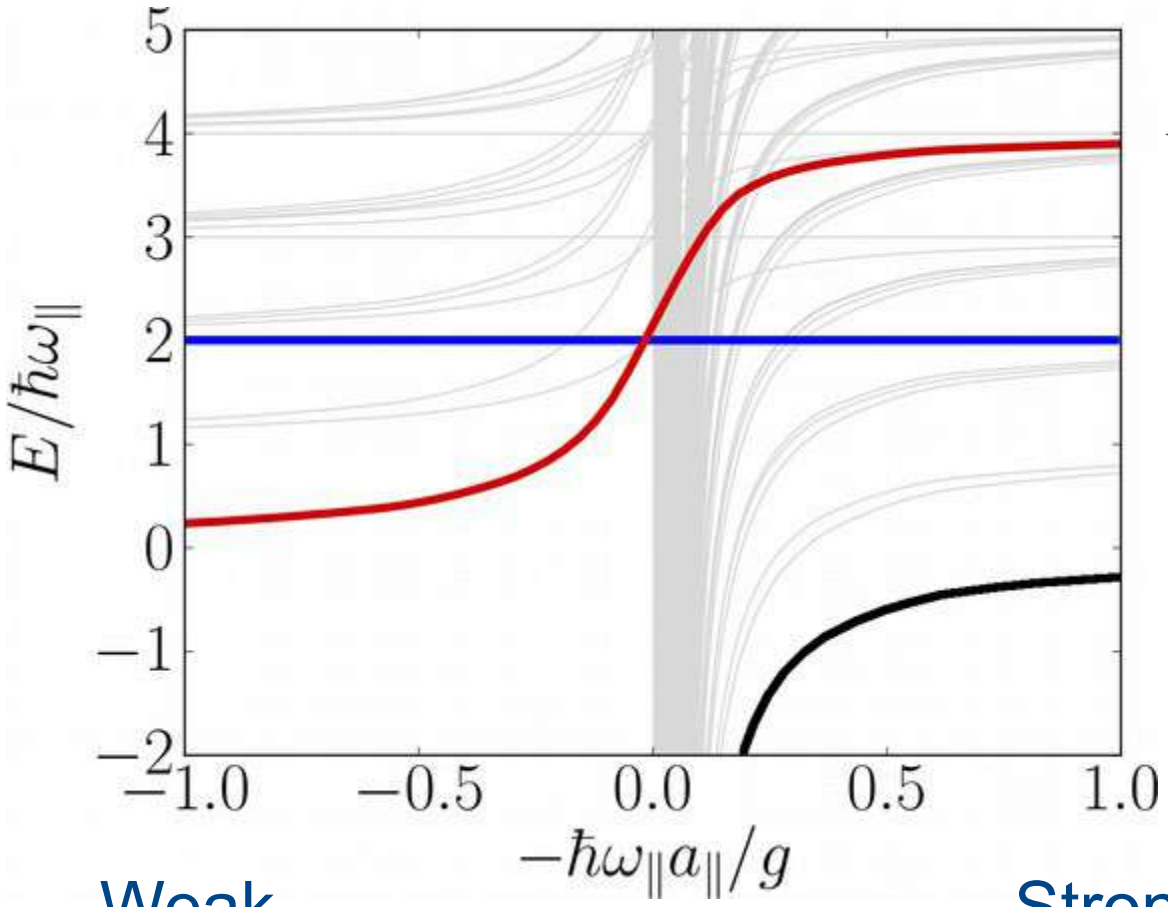


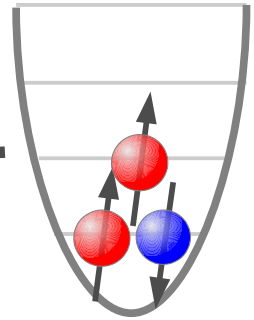


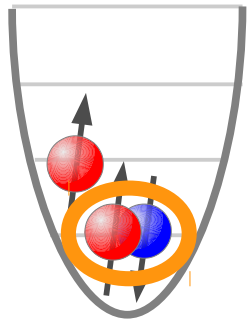
Weak

Strong

Three-atom bound state



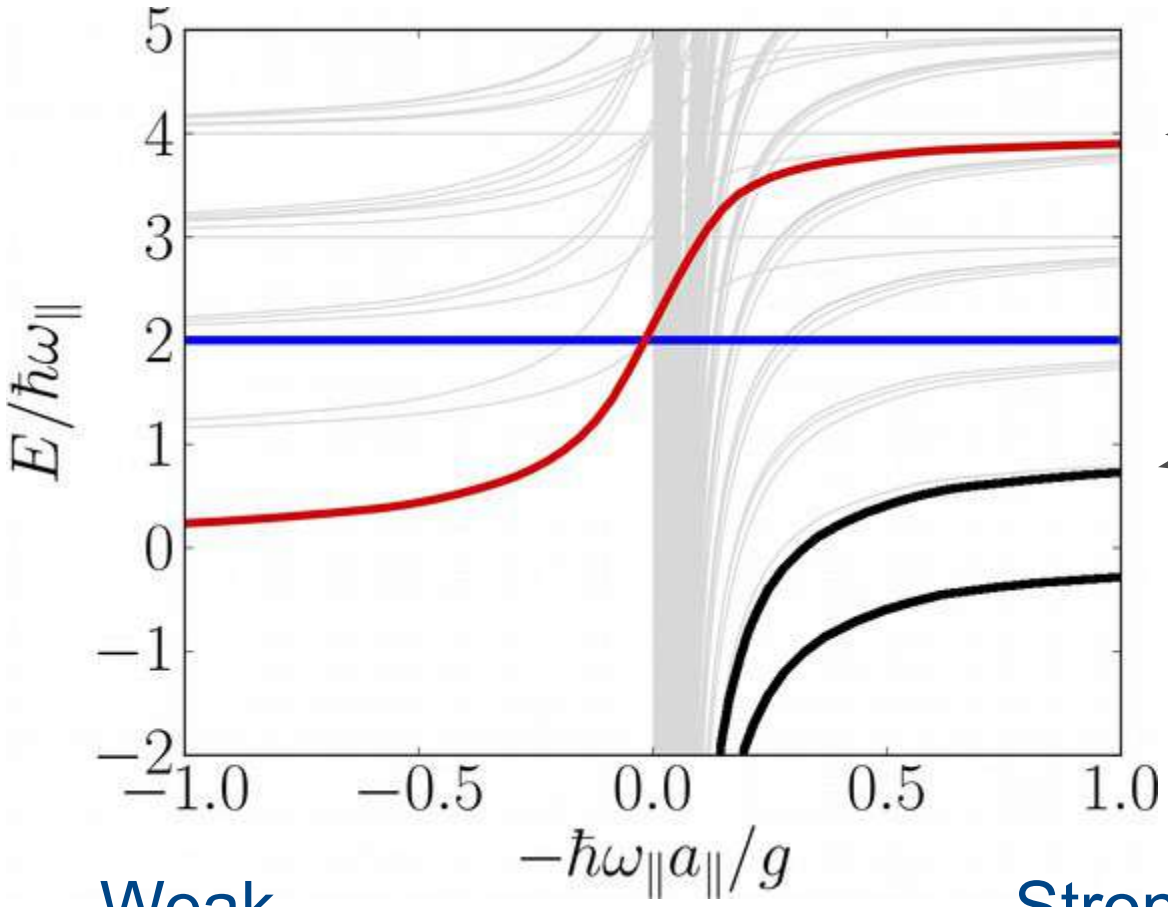


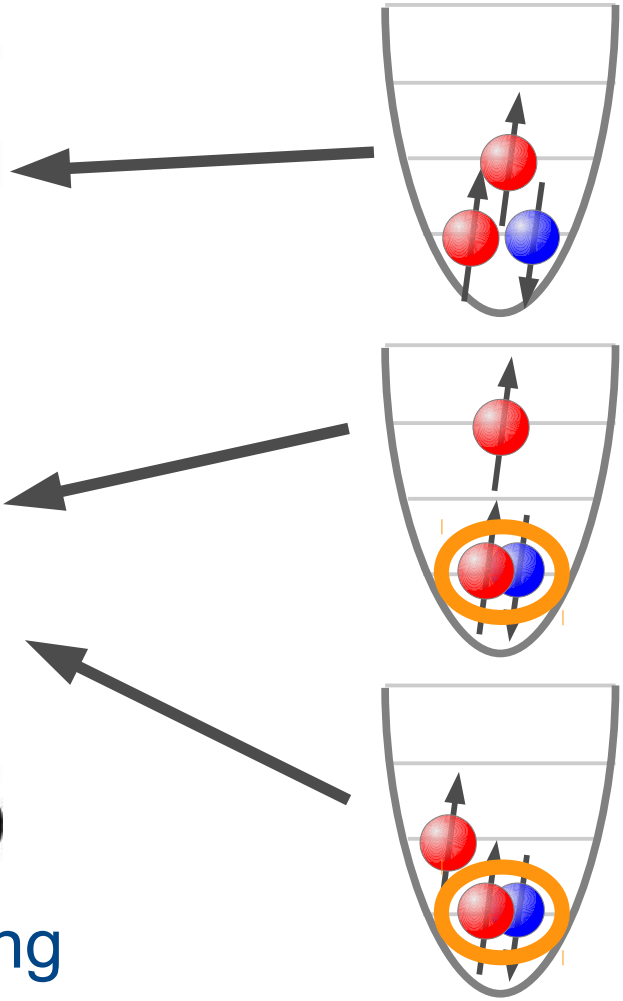


Weak

Strong

Three-atom bound state

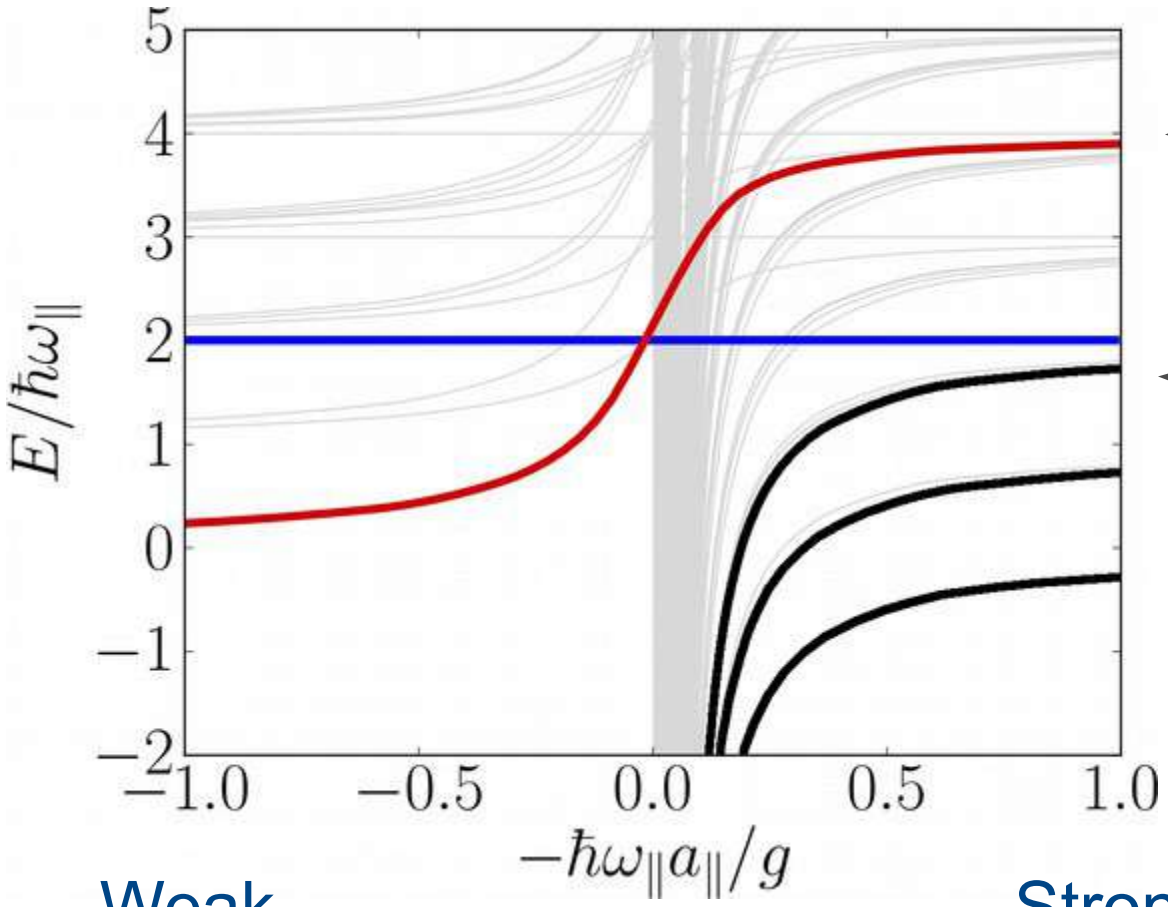


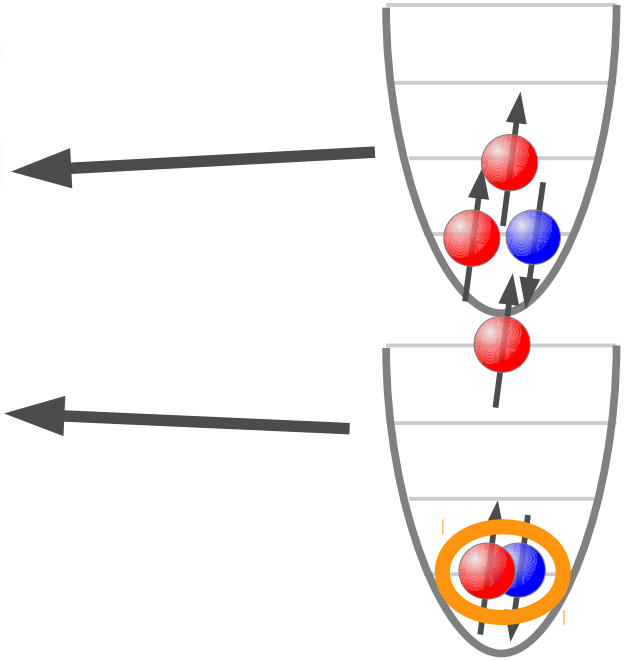


Weak

Strong

Three-atom bound state

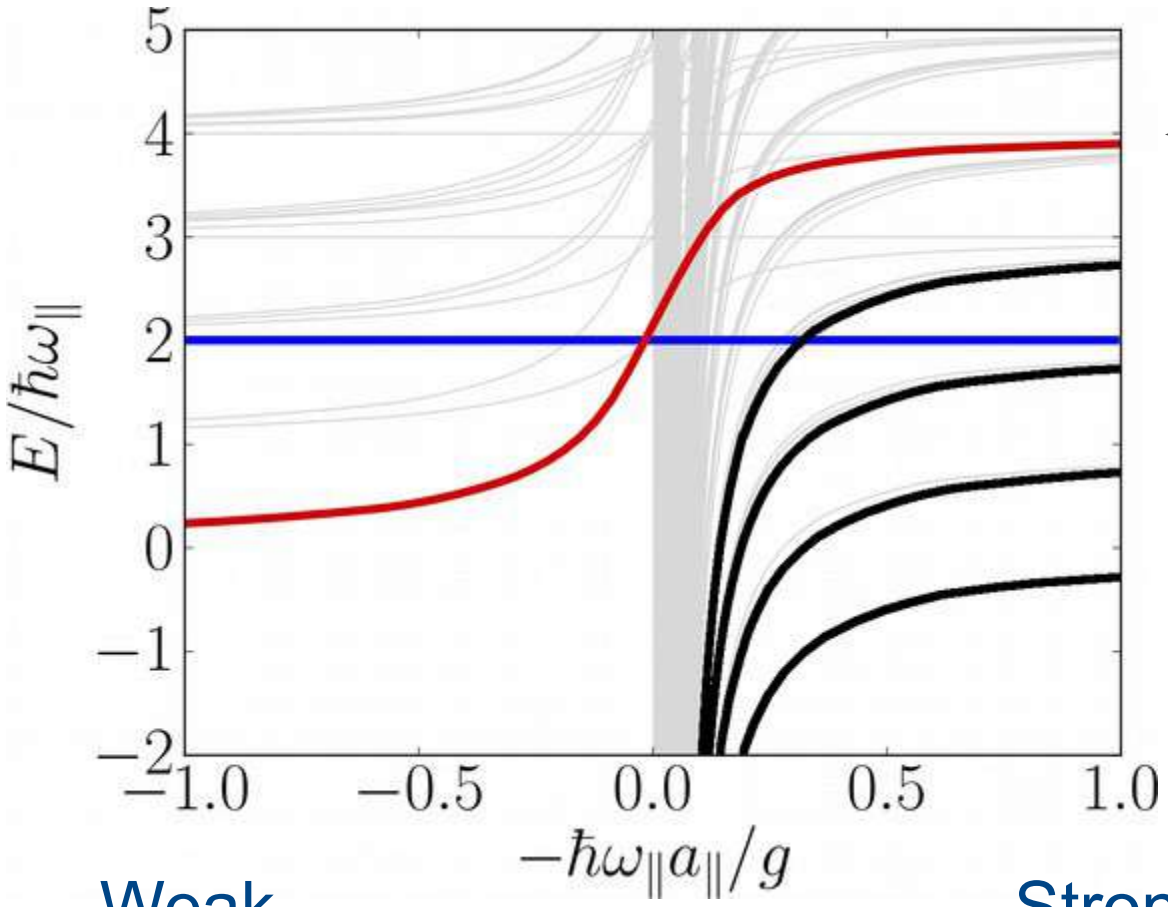


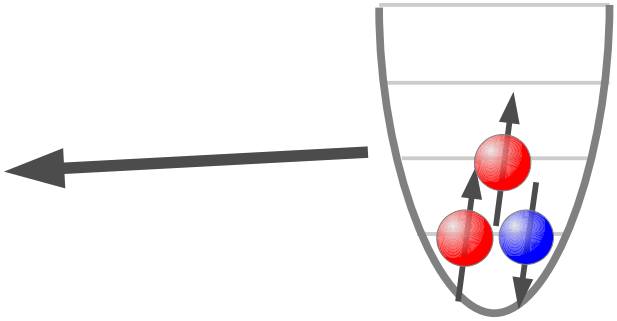


Weak

Strong

Three-atom bound state

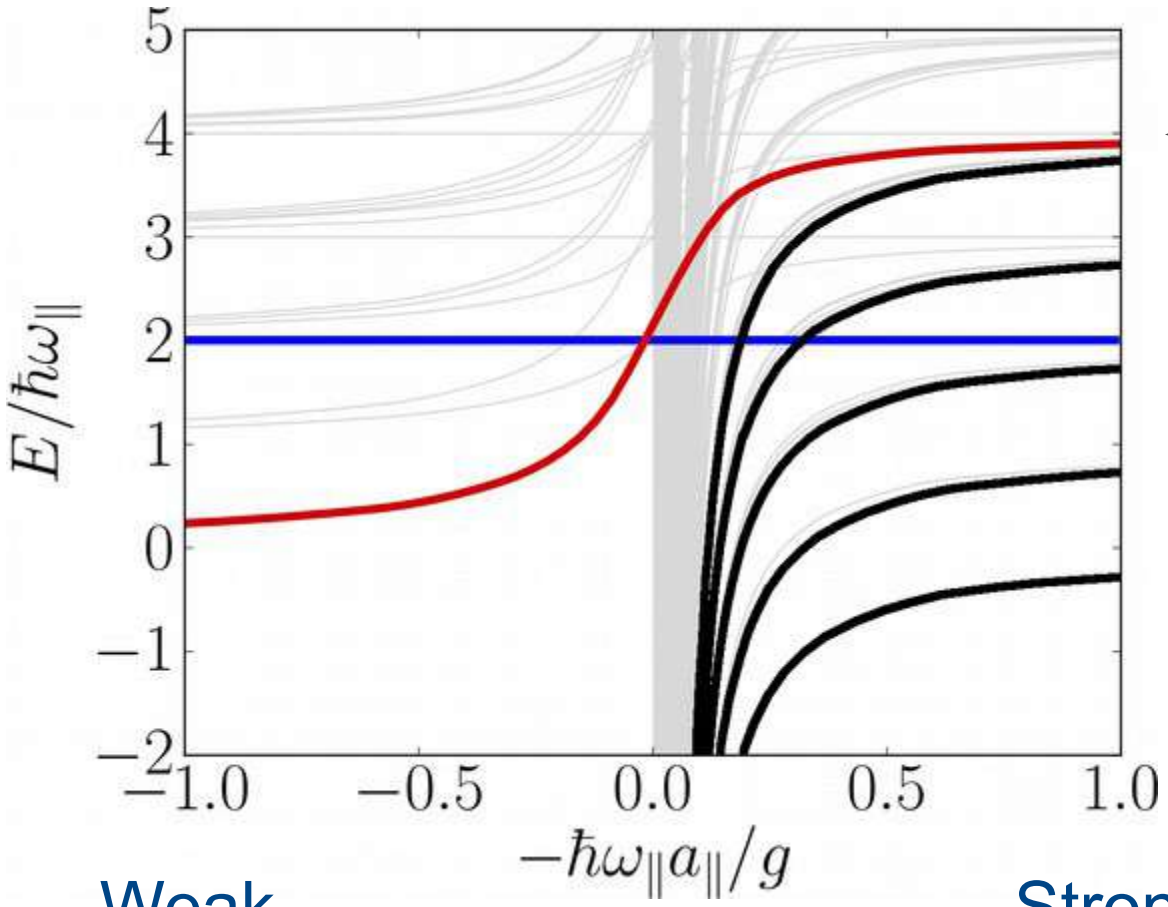


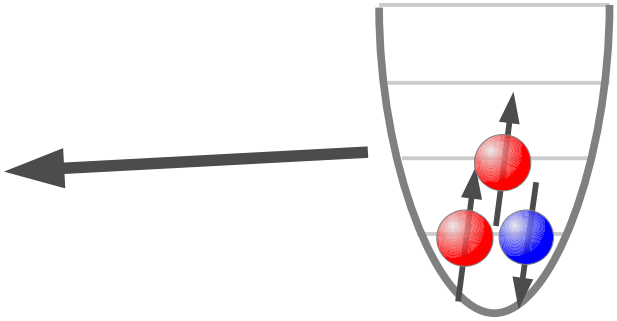


Weak

Strong

Three-atom bound state

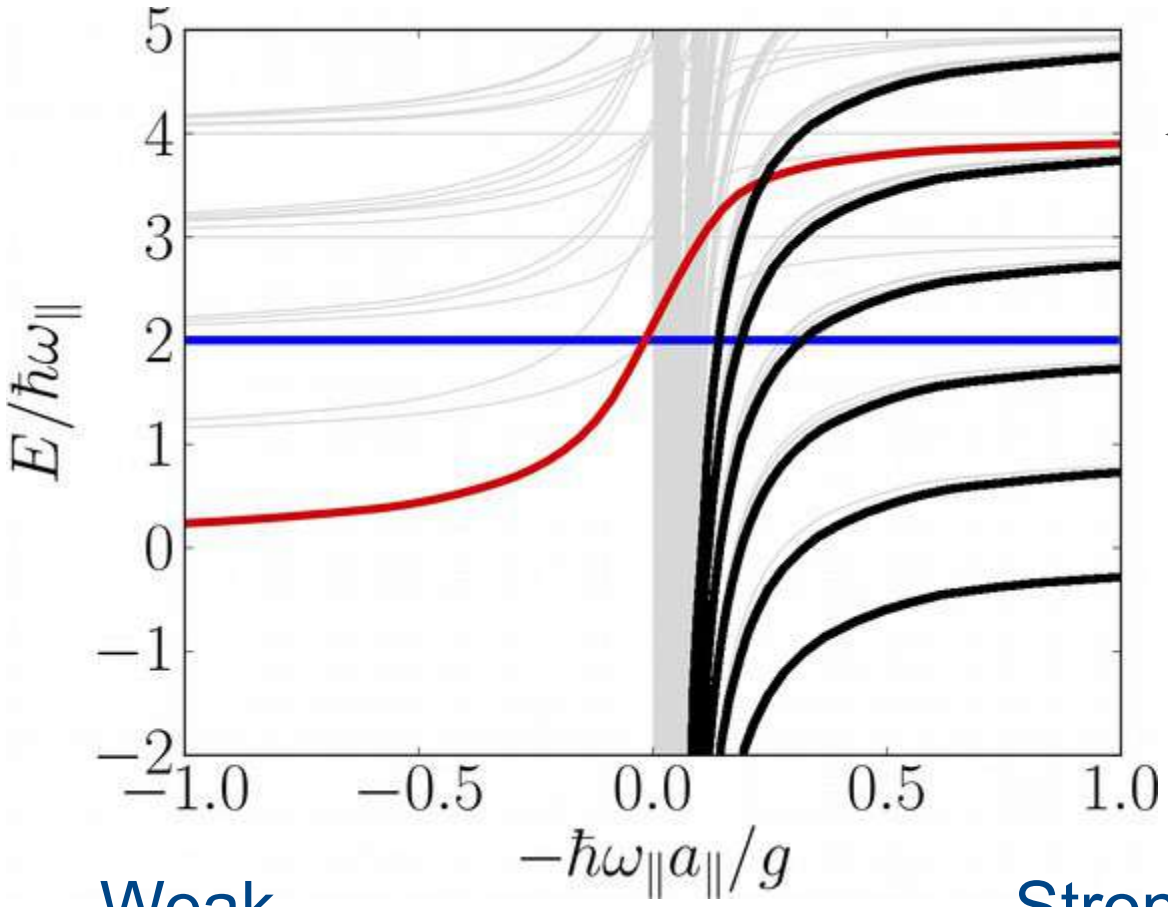


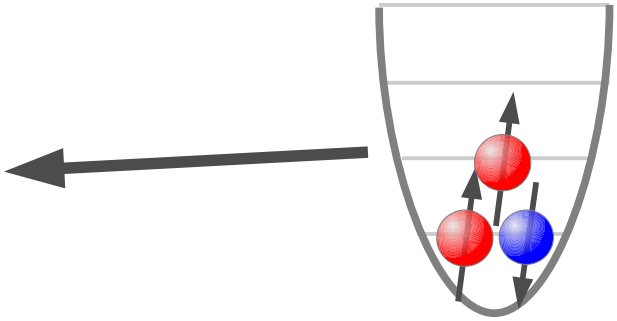


Weak

Strong

Three-atom bound state

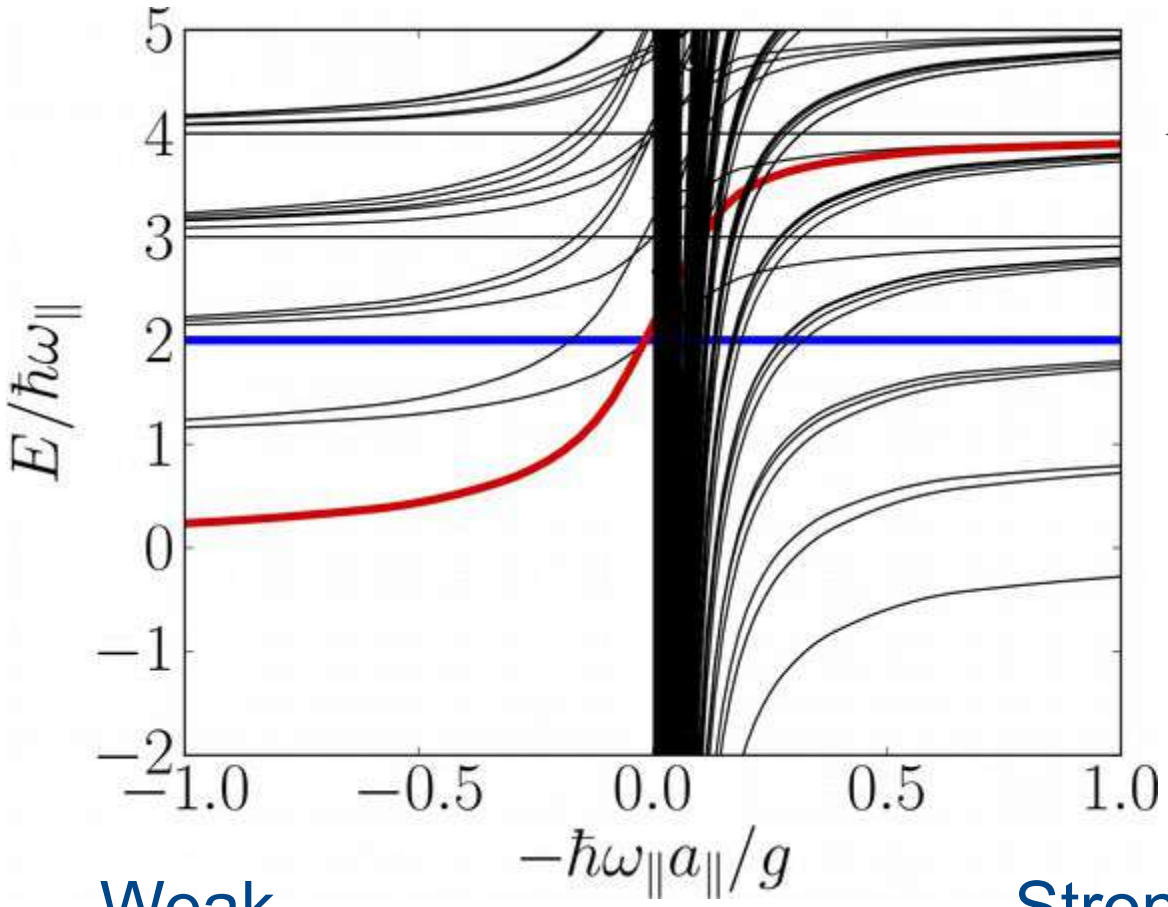


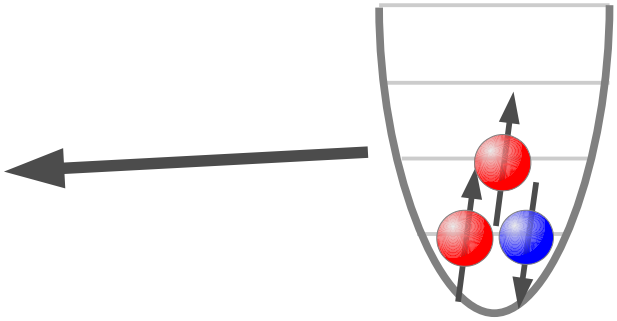


Weak

Strong

Three-atom bound state

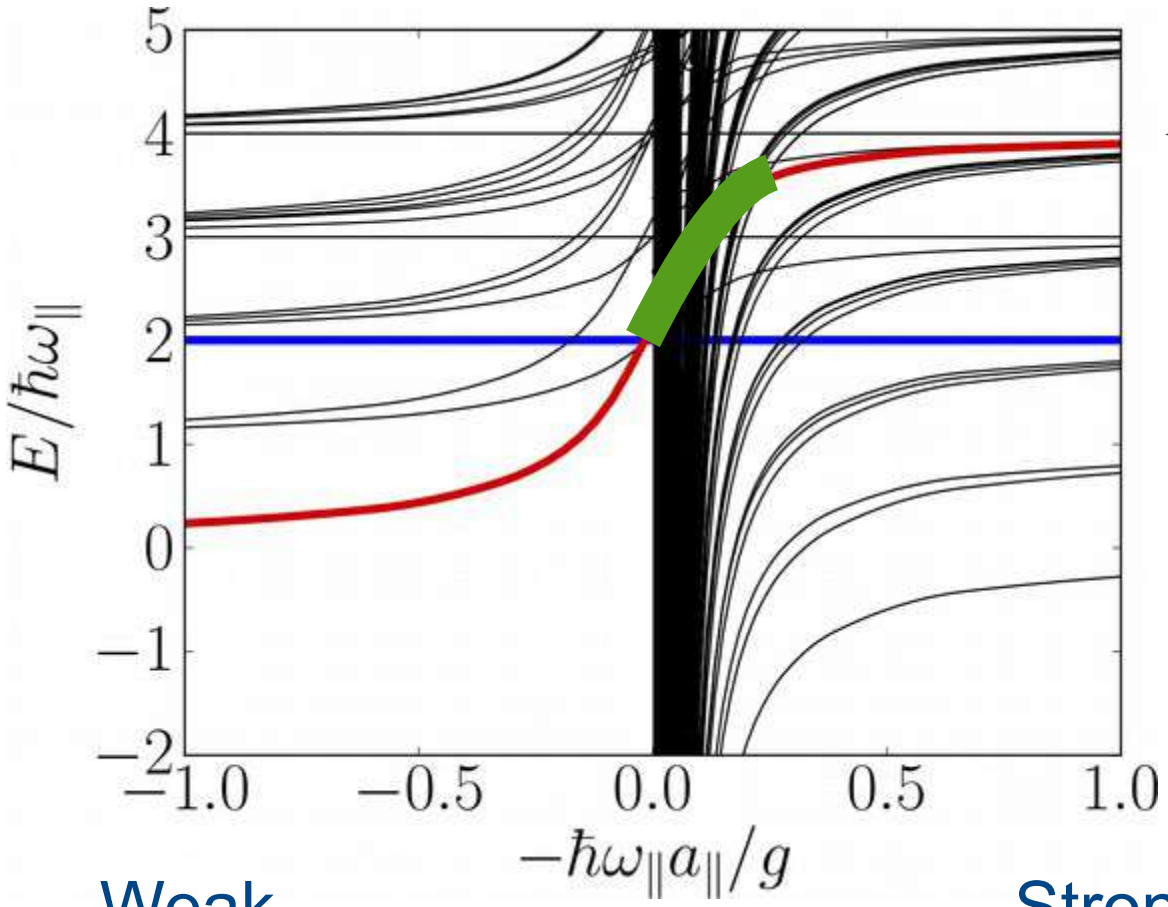


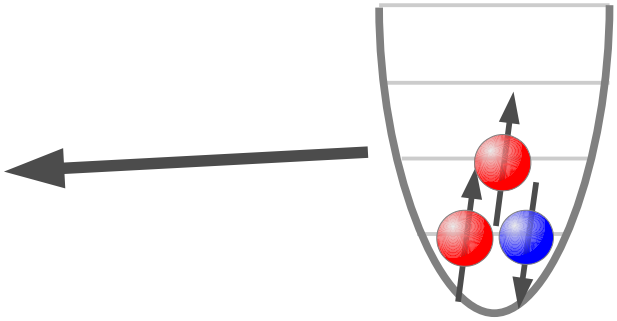


Weak

Strong

Three-atom bound state

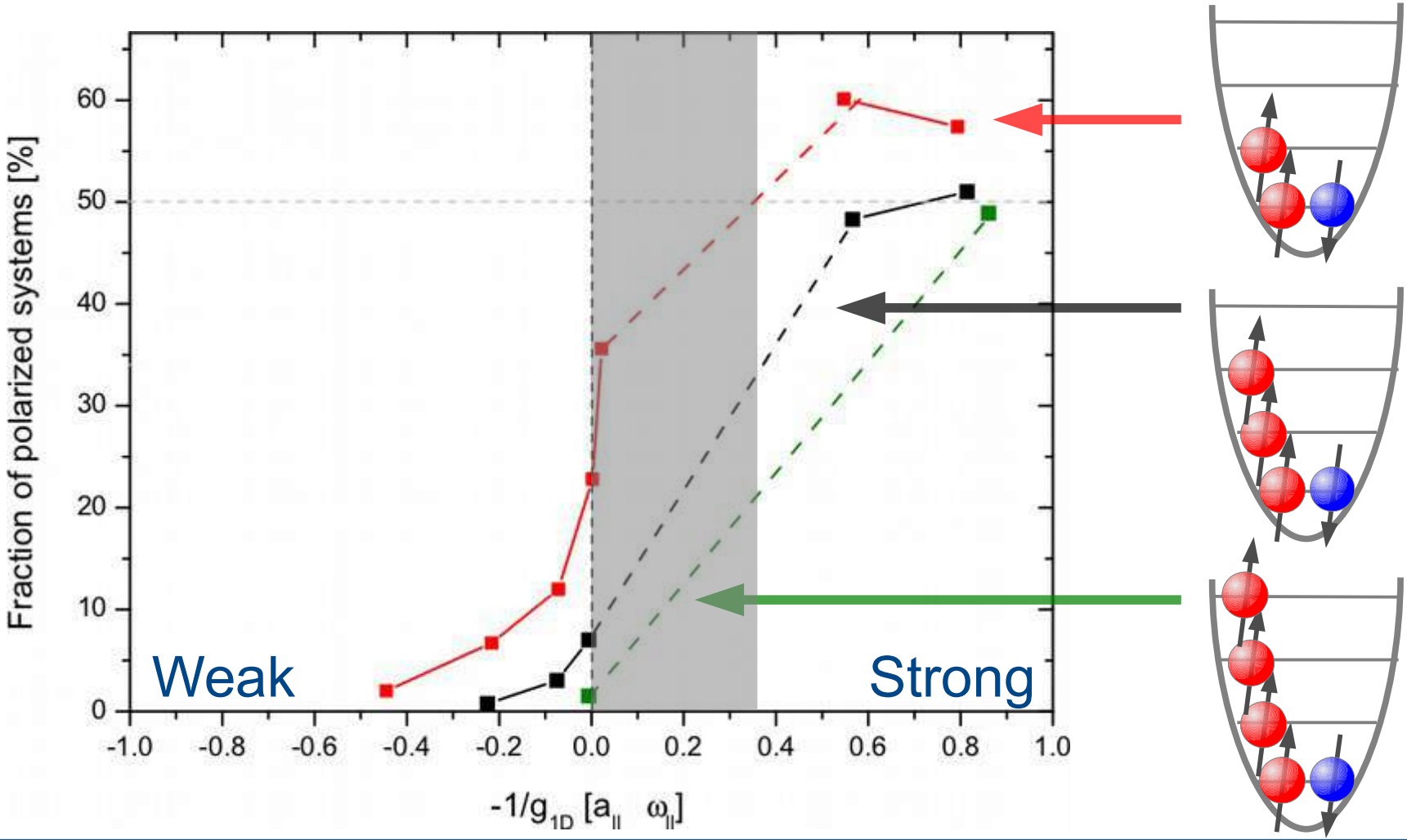




Weak

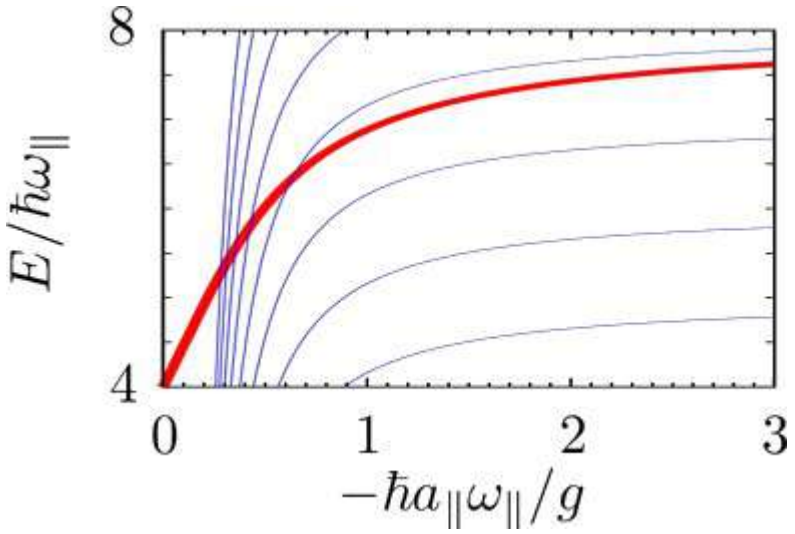
Strong

Tunneling probability



Band crossings

$n_\uparrow n_\downarrow = 4$



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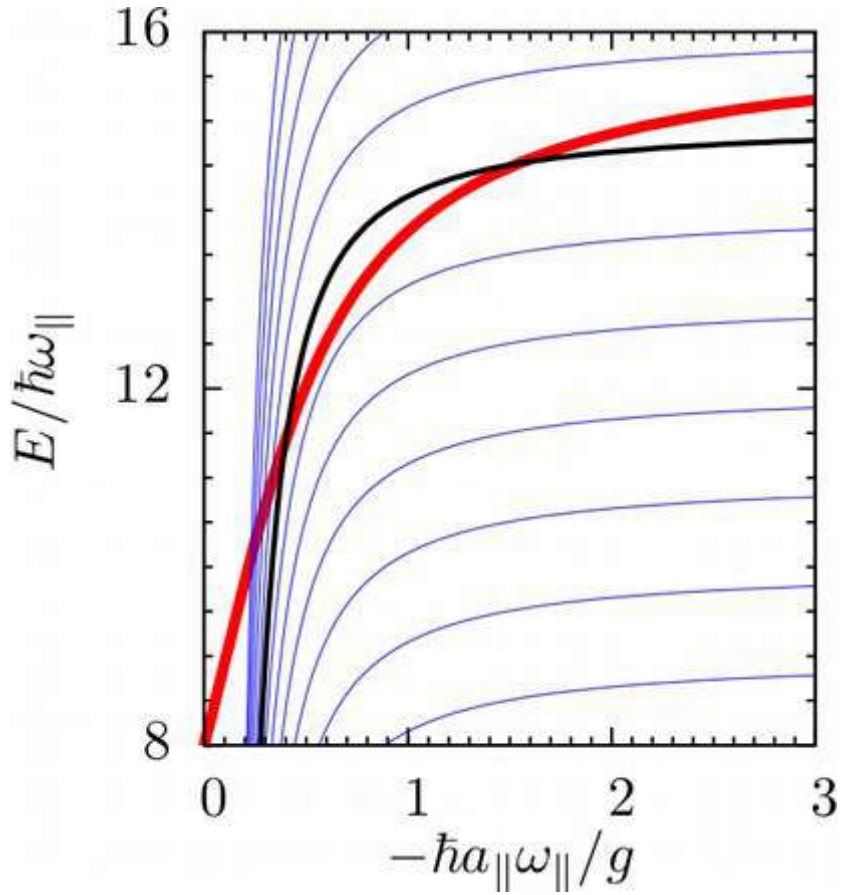
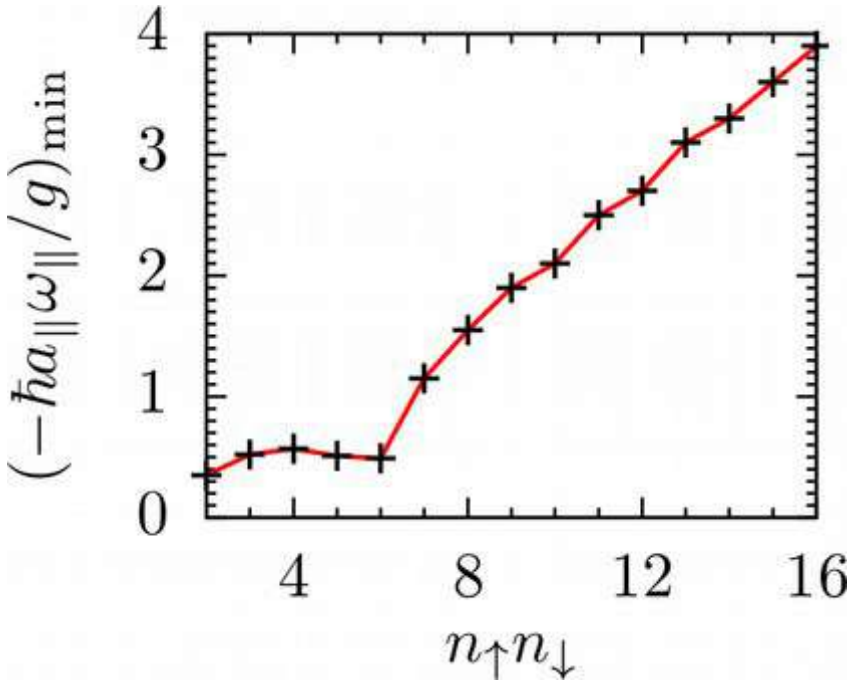
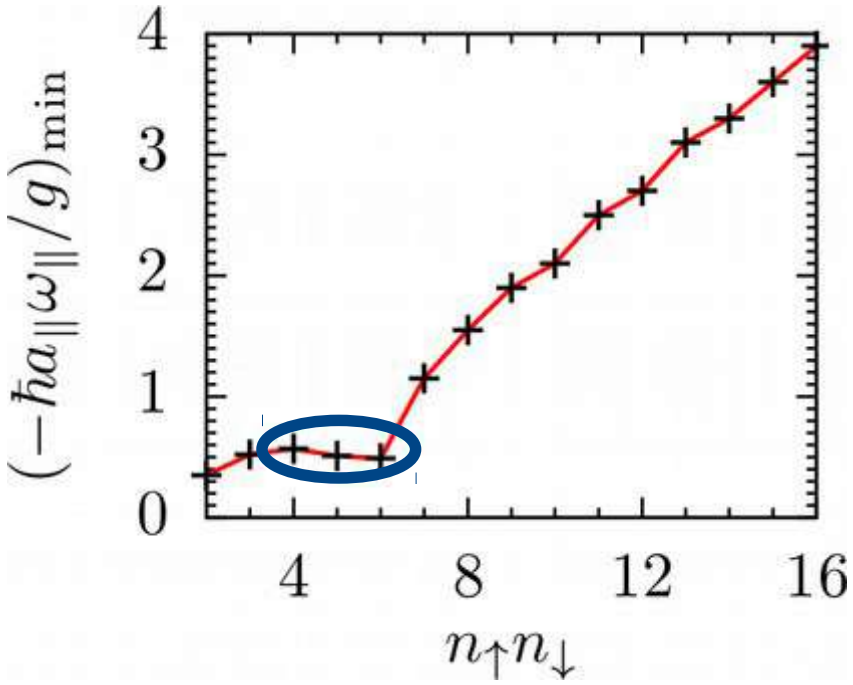


Band crossings



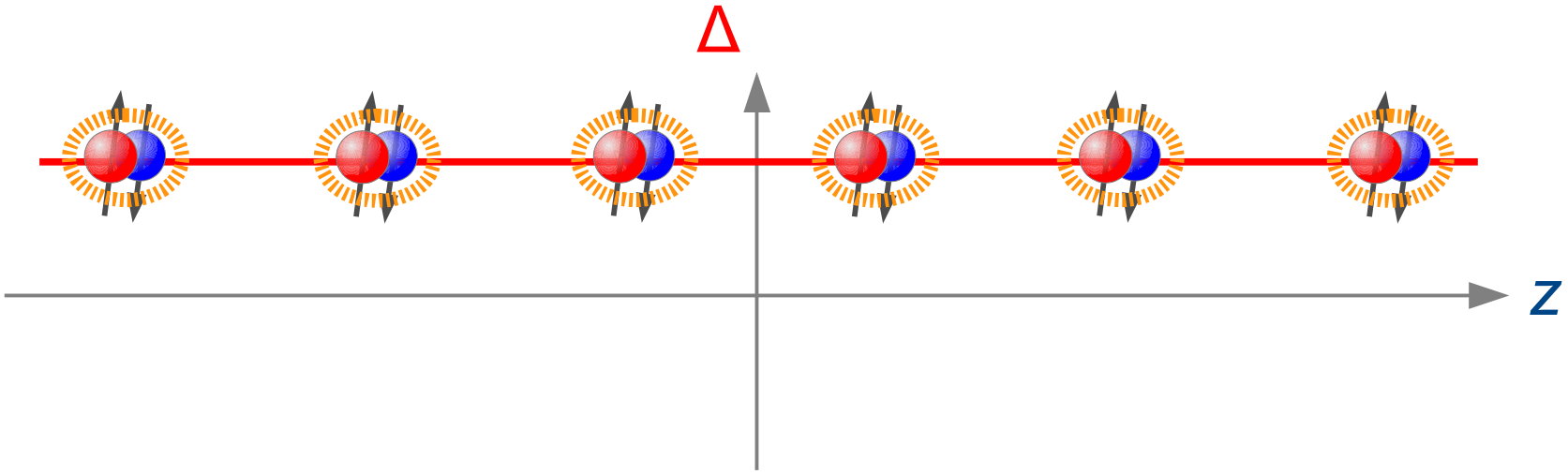
Band crossings



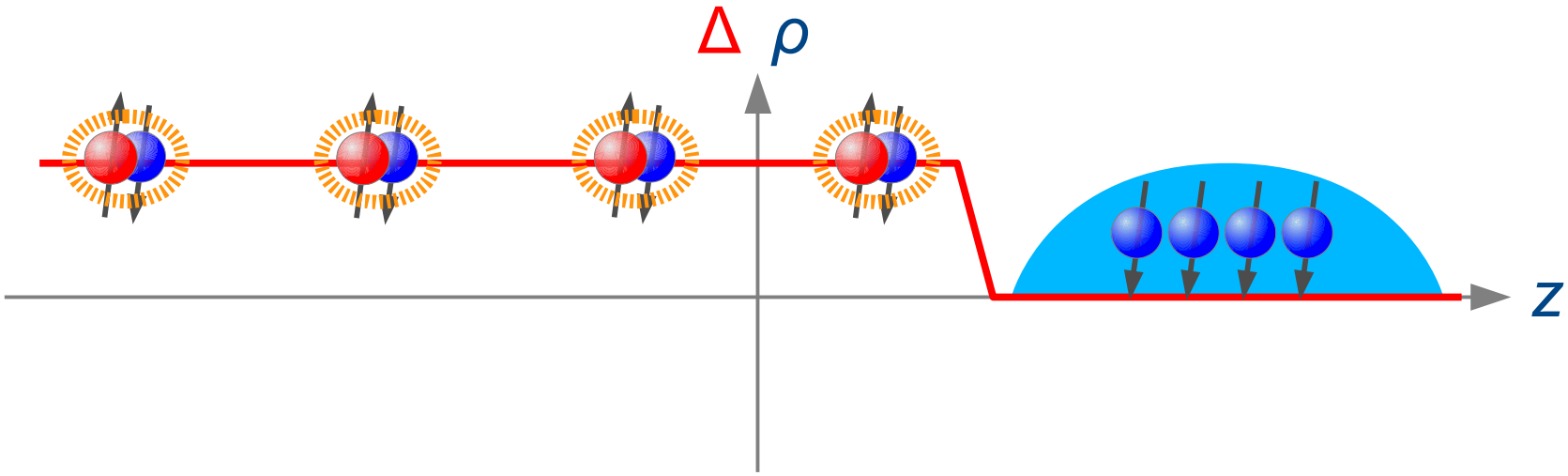
BCS Hamiltonian

$$\hat{H} = -\frac{\nabla^2}{2} + g c^{\dagger}_{\vec{p}\uparrow} c^{\dagger}_{-\vec{p}\downarrow} c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow} + V(\vec{r})$$

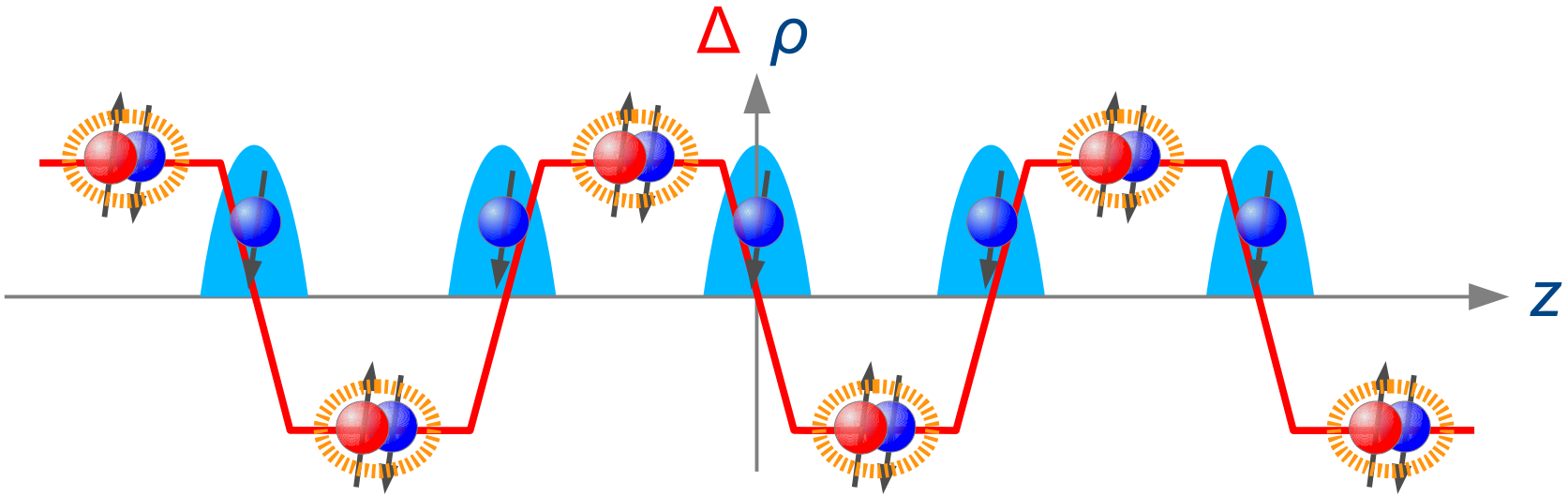
Inhomogeneous pairing



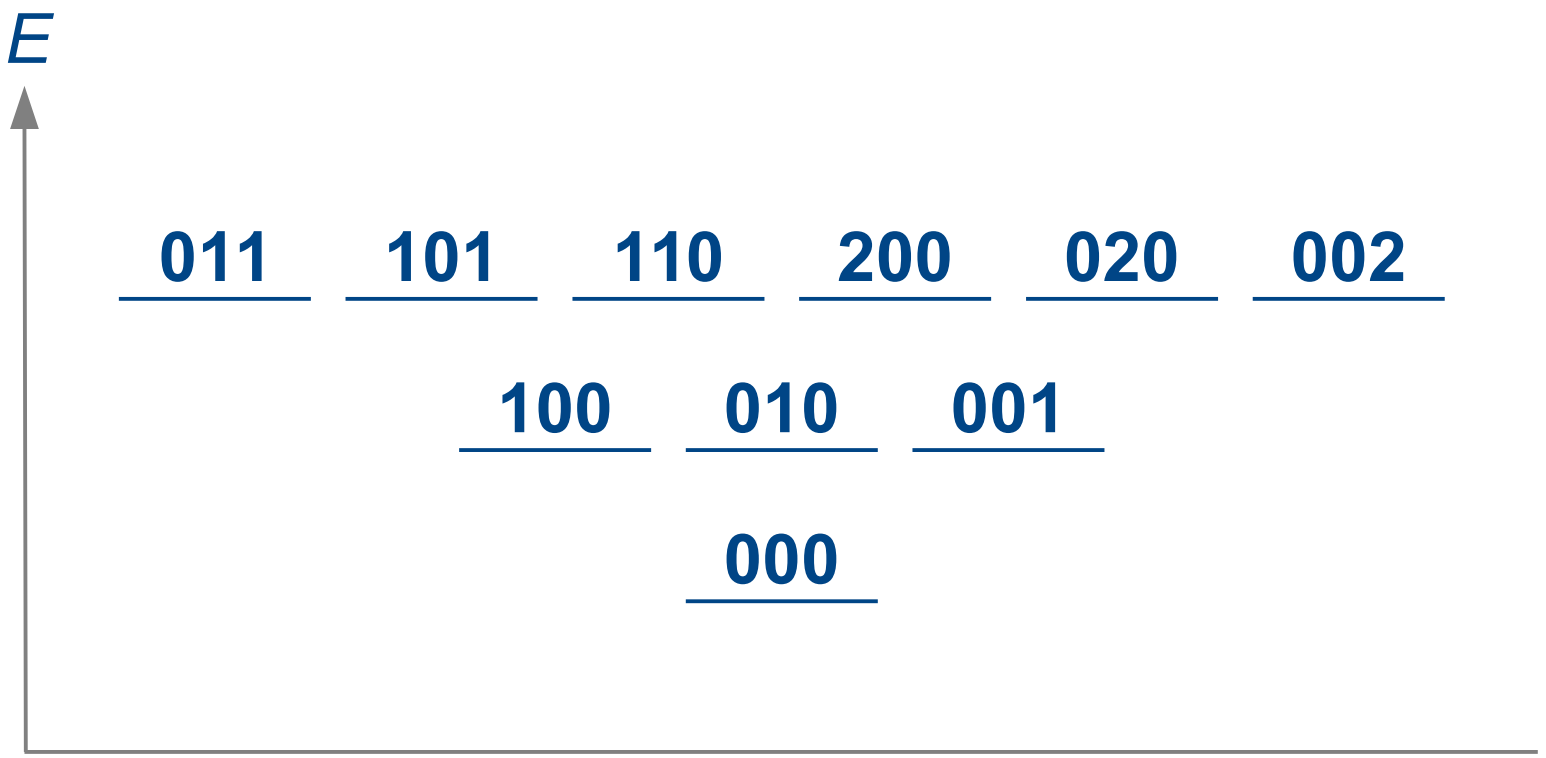
Inhomogeneous pairing



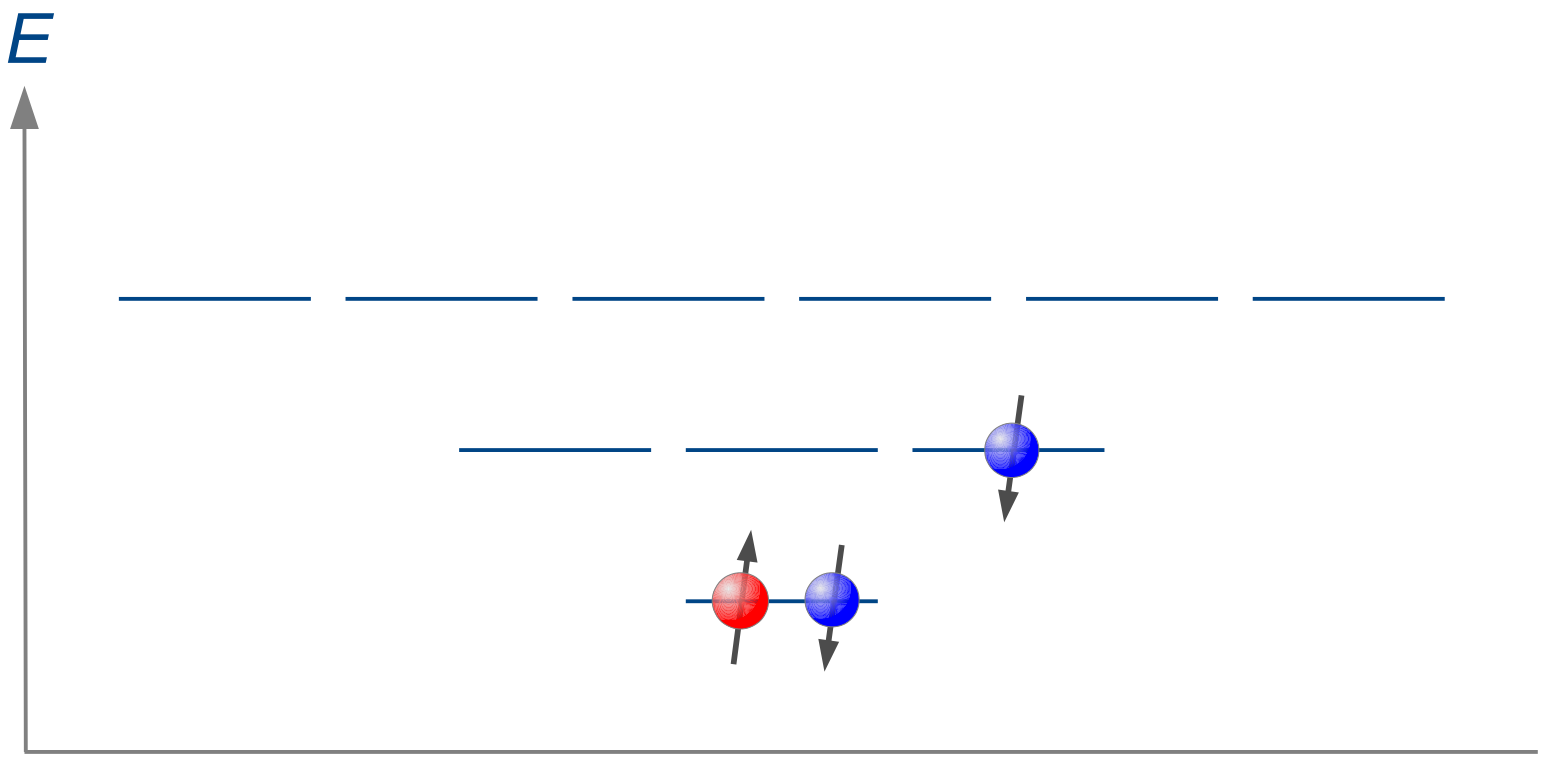
Inhomogeneous pairing



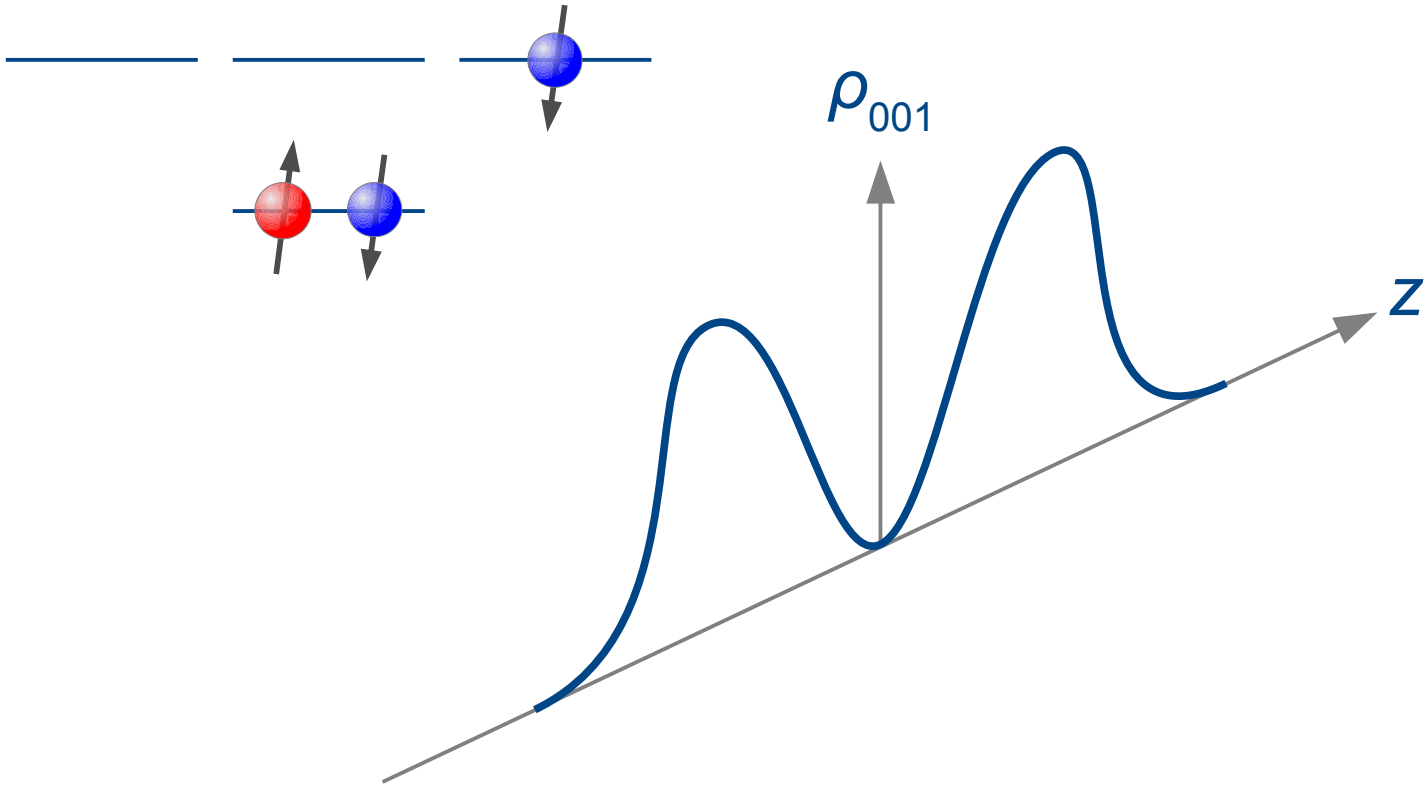
Inhomogeneous pairing



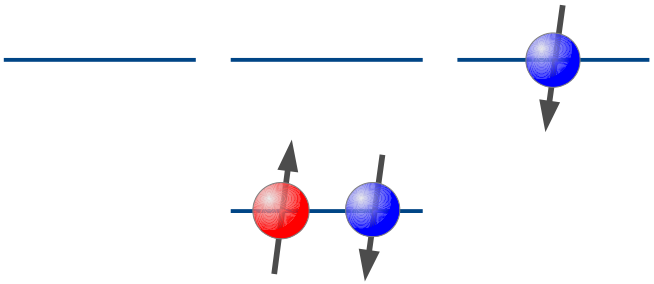
Inhomogeneous pairing

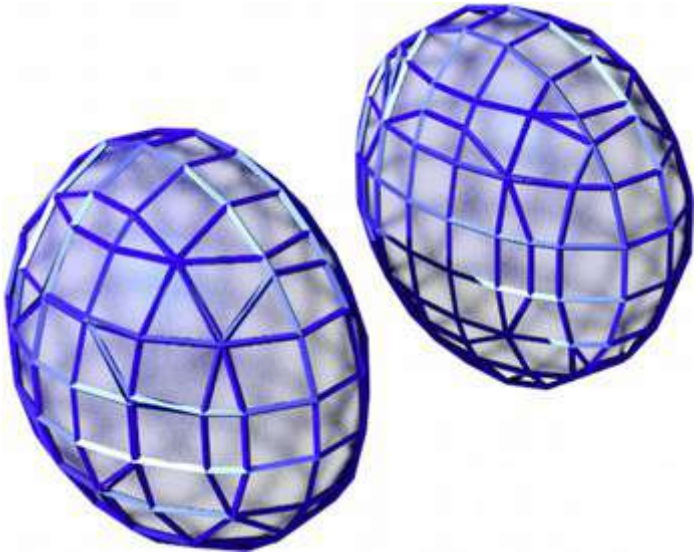


Inhomogeneous pairing

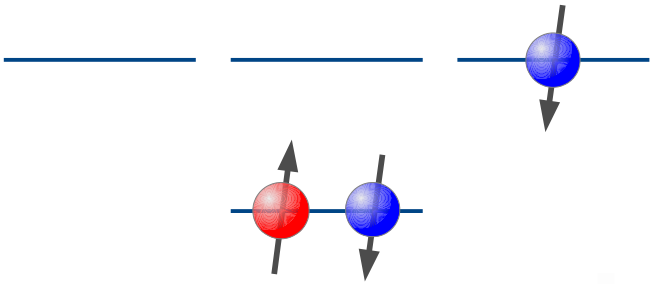


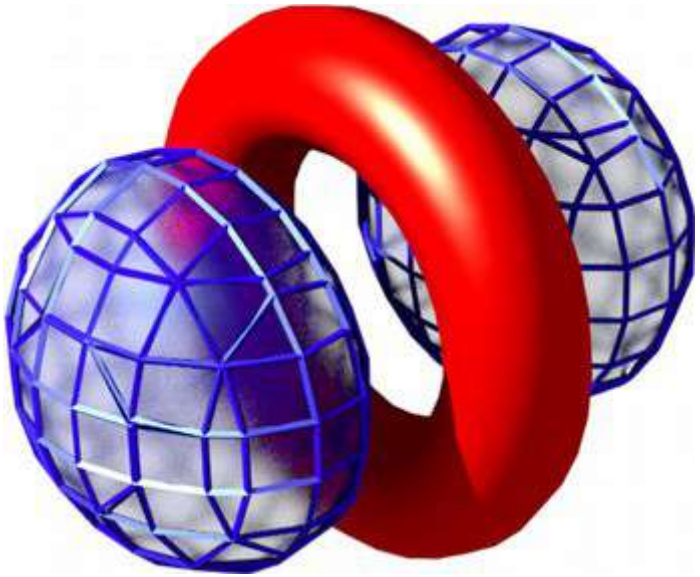
Inhomogeneous pairing



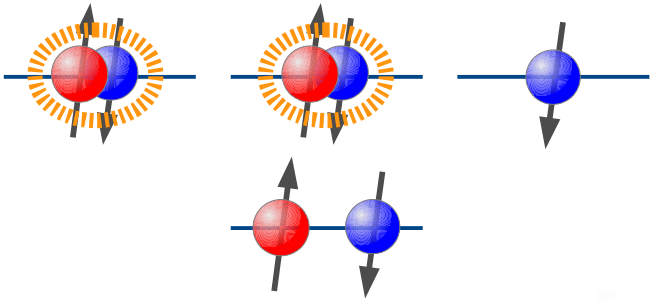


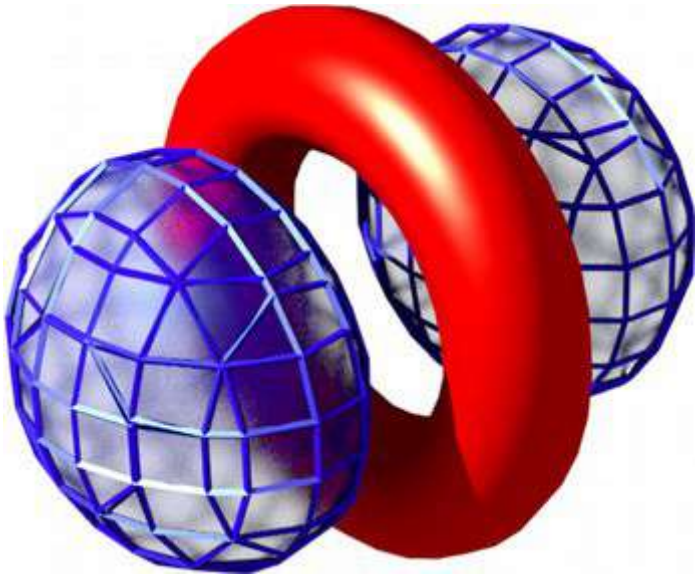
Inhomogeneous pairing



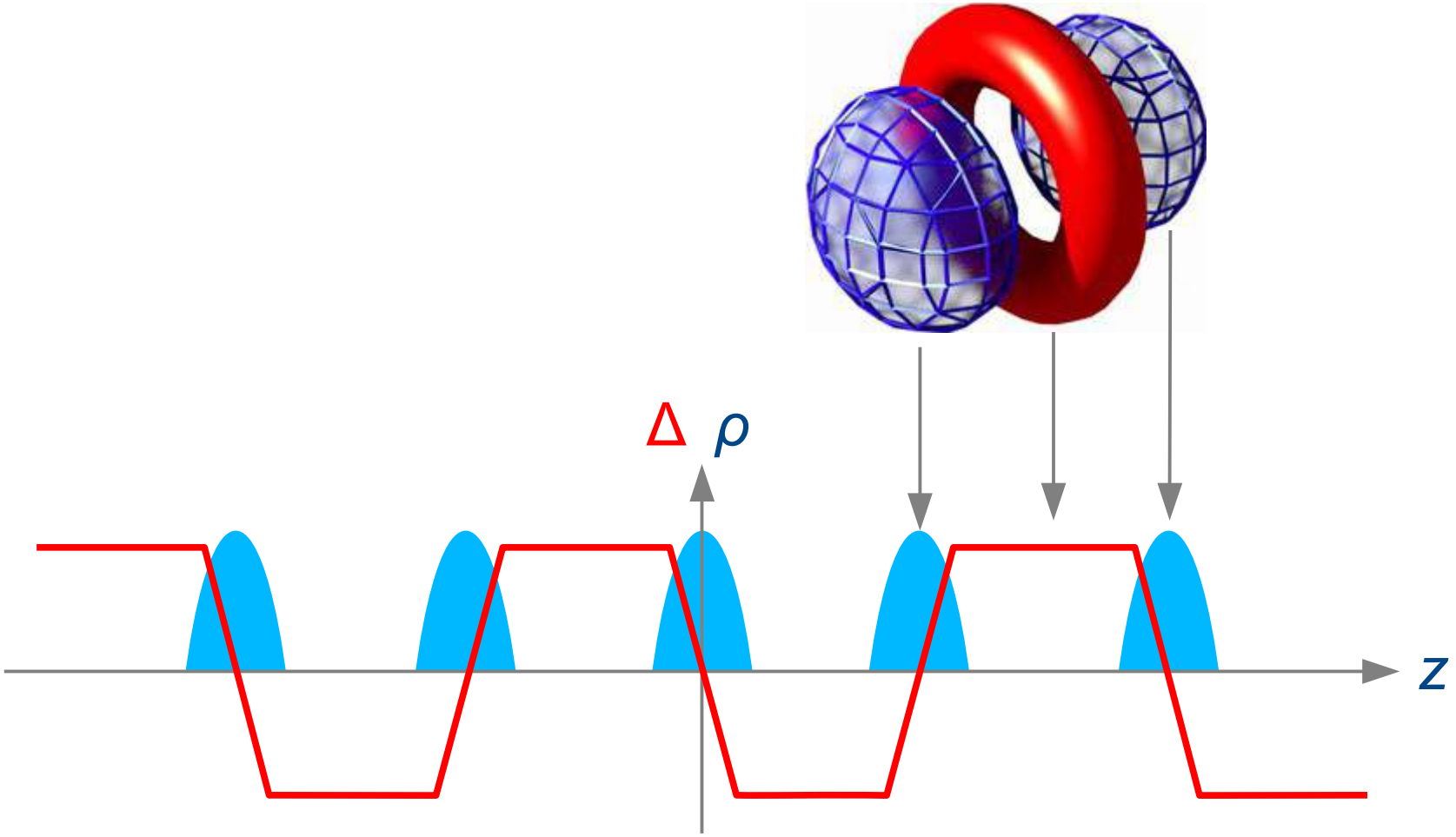


Inhomogeneous pairing





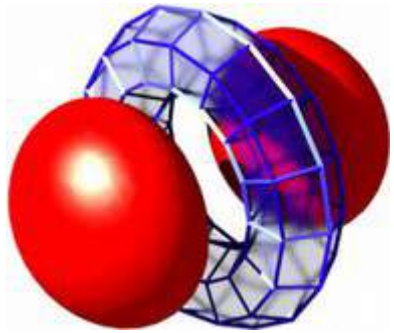
Inhomogeneous pairing



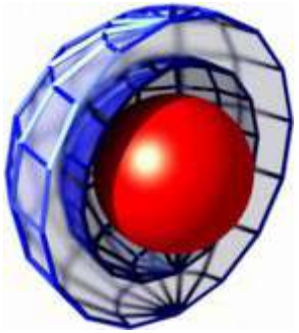
Inhomogeneous pairing

(2,1) (3,2) (4,3)

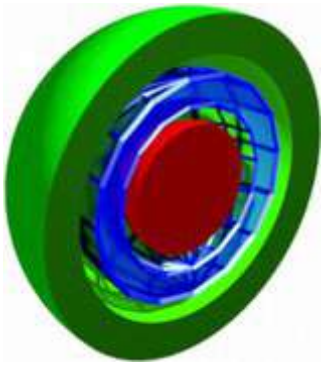
(3,1) (4,2)



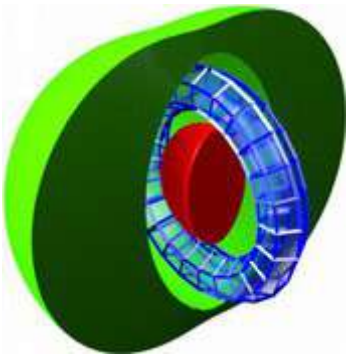
(4,1)



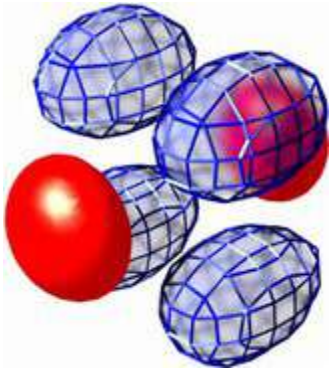
$(2,1)^s$ (5,1)



$(2,1)^d$ (5,2)



$(2,1)^g$



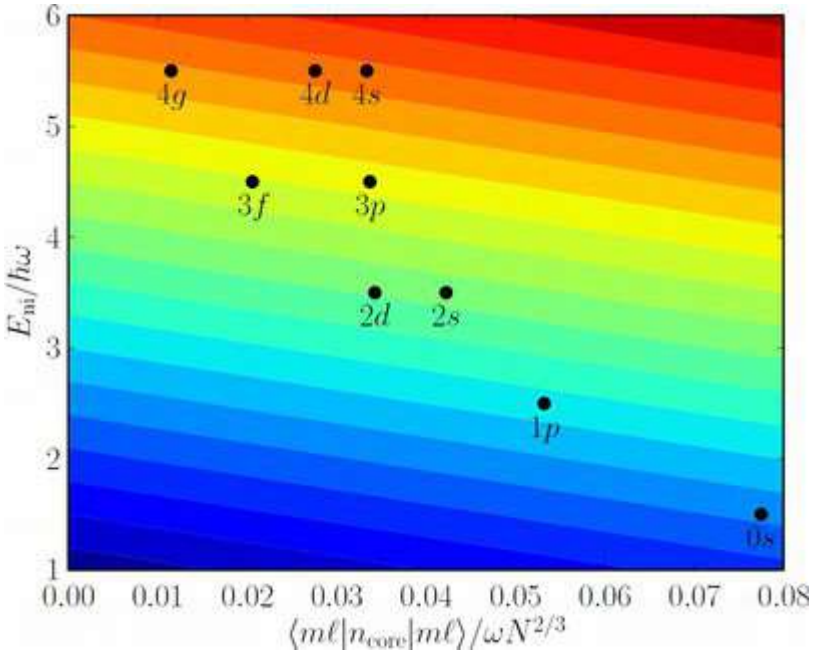
Summary

A few-fermion system provides insight into many-body physics

Discretization of energy levels means that losses occur in narrow range of interaction strengths

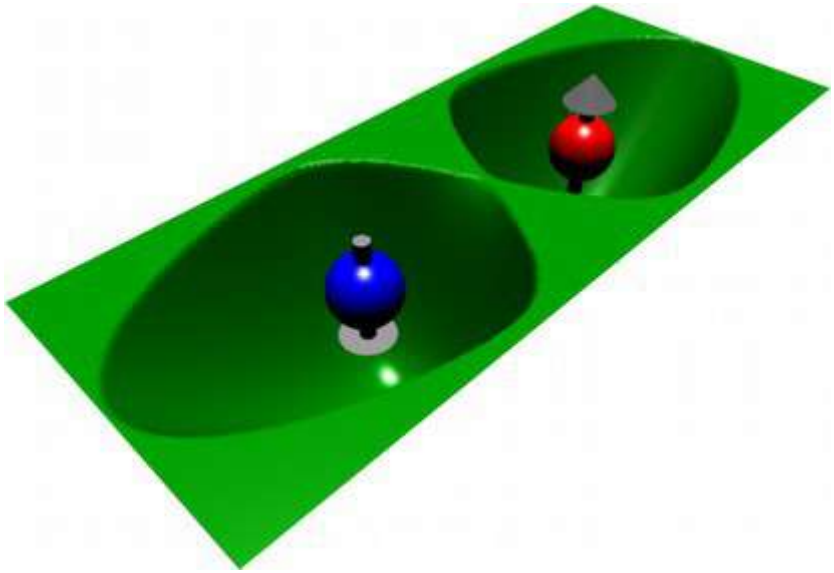
Observation of Fermi surface and magnetic correlations

Other phenomena



Hund's rules

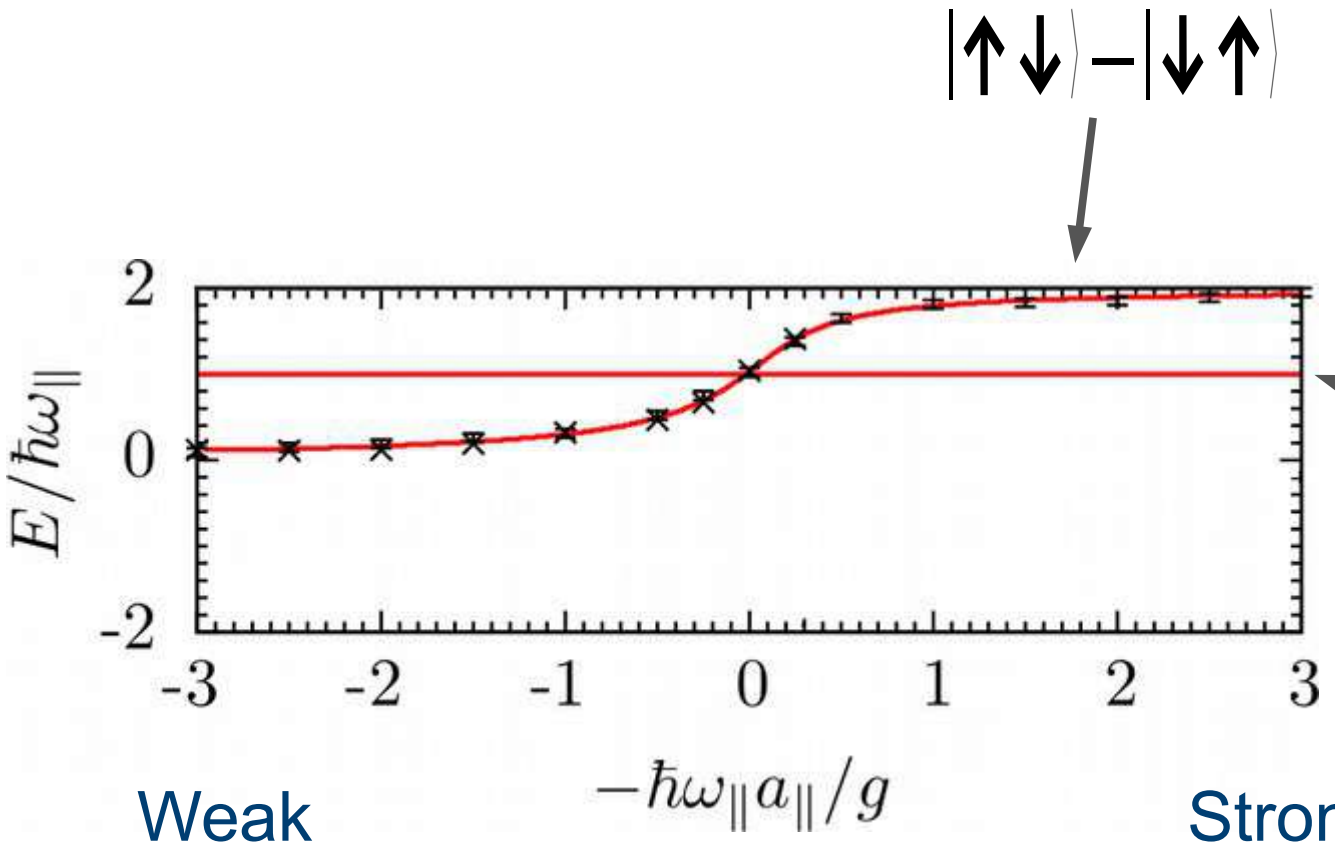
P.O. Bugnion, J.A. Lofthouse & GJC



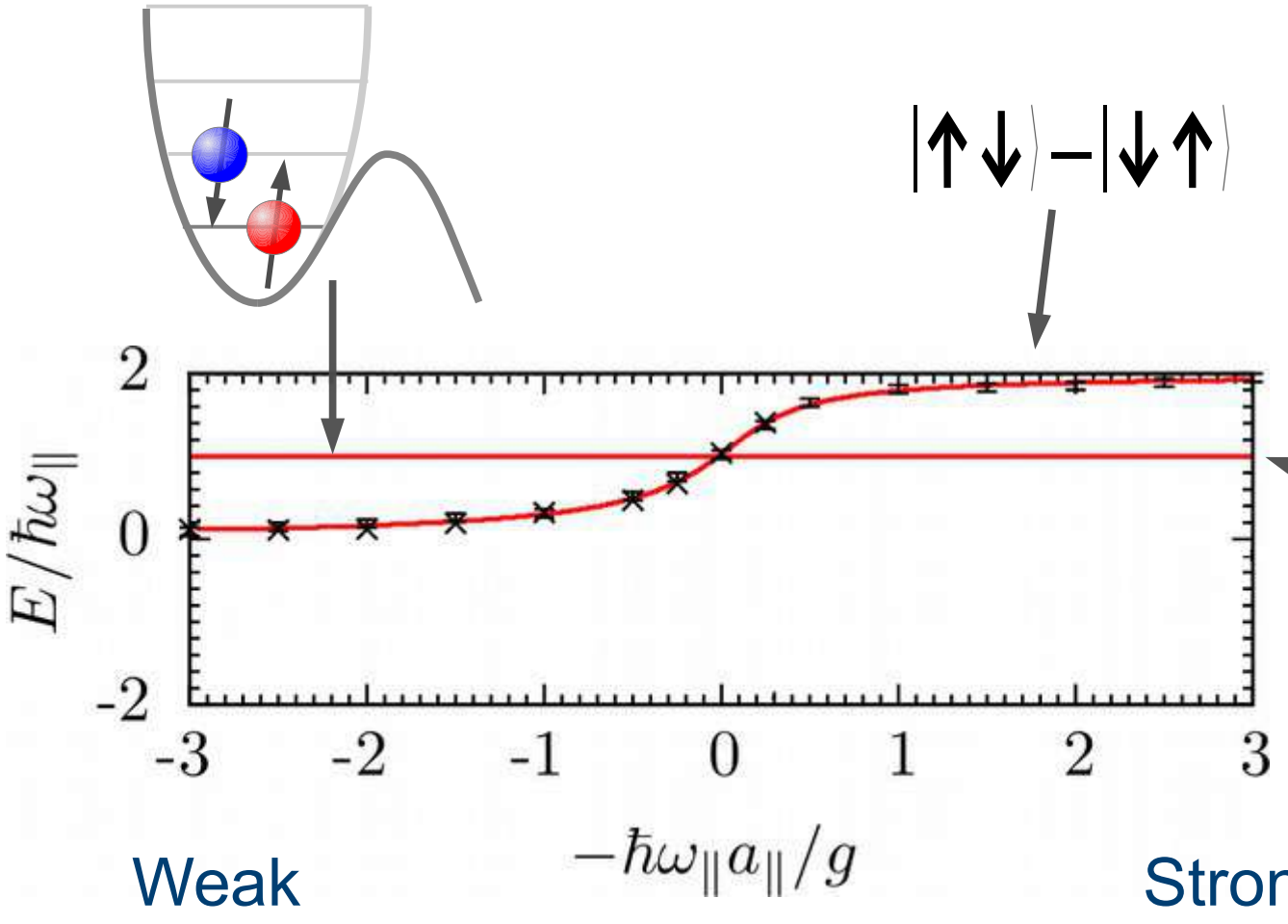
Exchange interactions

P.O. Bugnion & GJC
Phys. Rev. A **88**, 013601 (2013)

Finding the missing probability

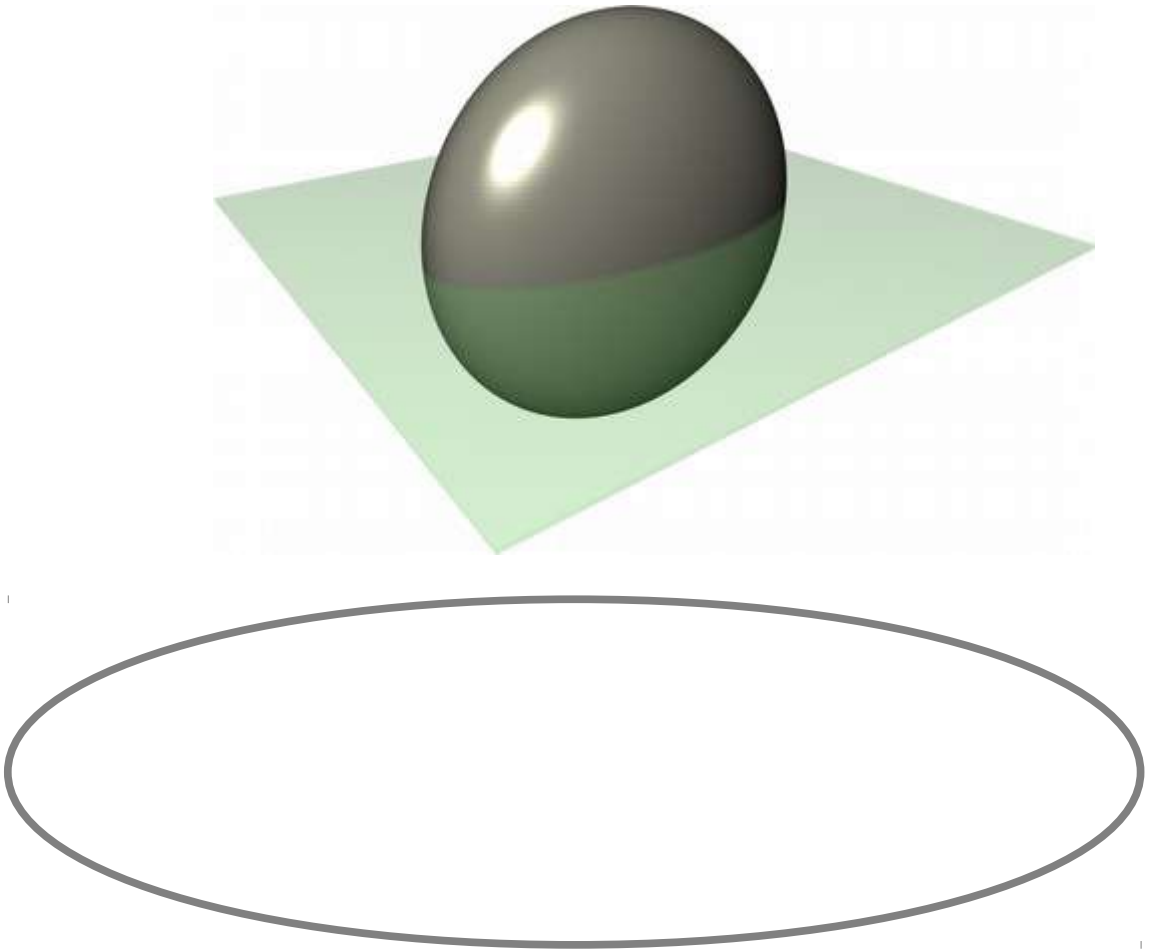


Finding the missing probability

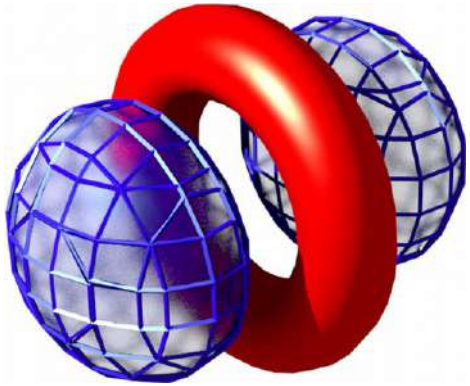


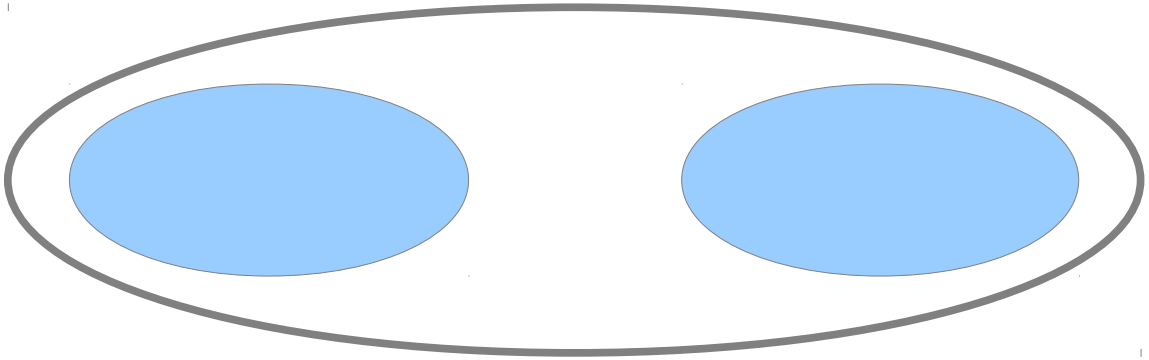
Inhomogeneous pairing

Inhomogeneous pairing

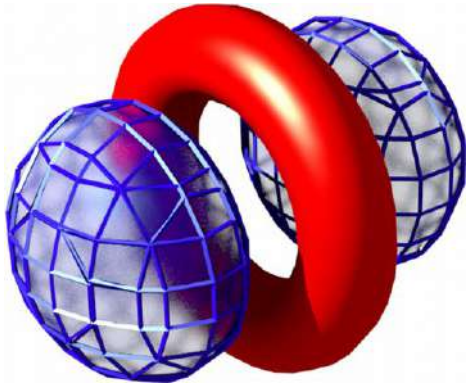


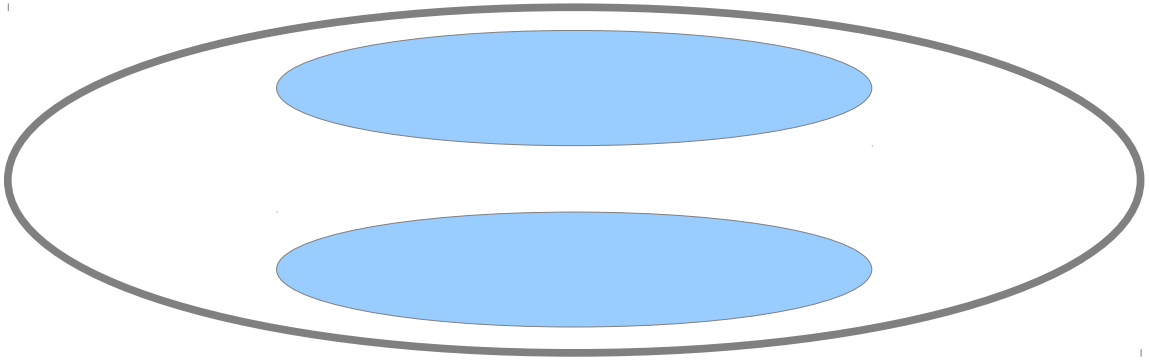
Inhomogeneous pairing



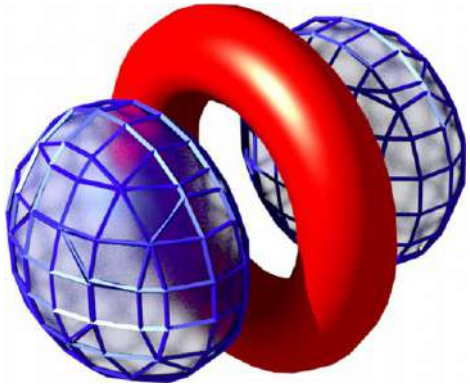


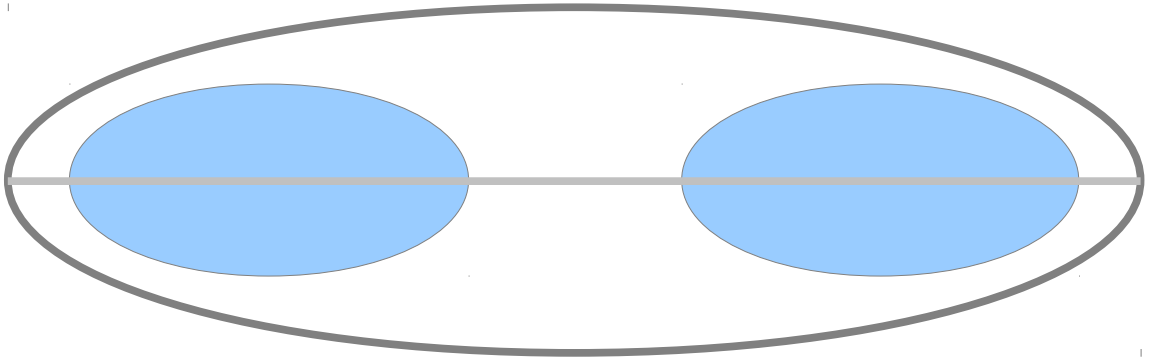
Inhomogeneous pairing





Inhomogeneous pairing





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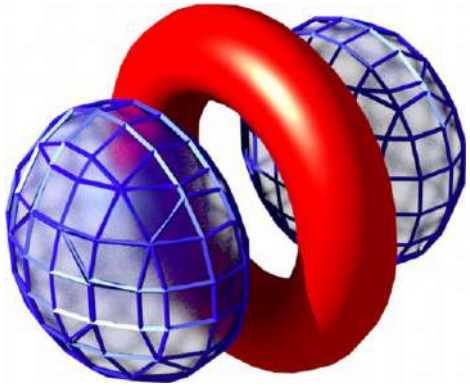
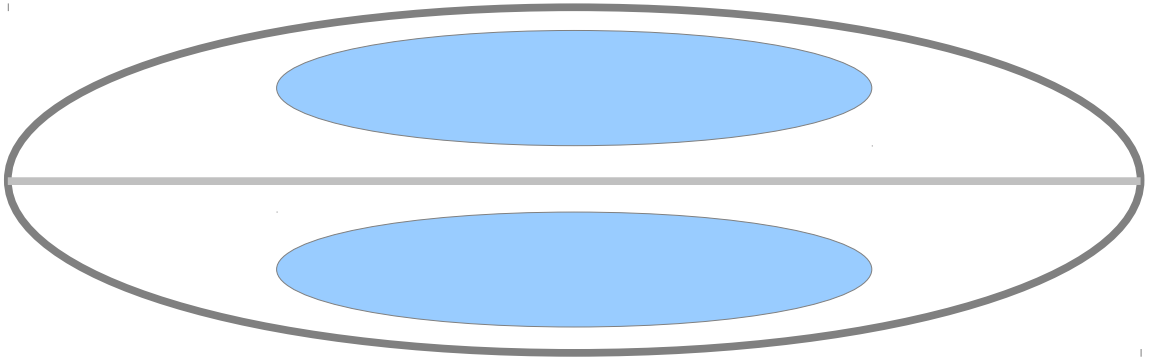
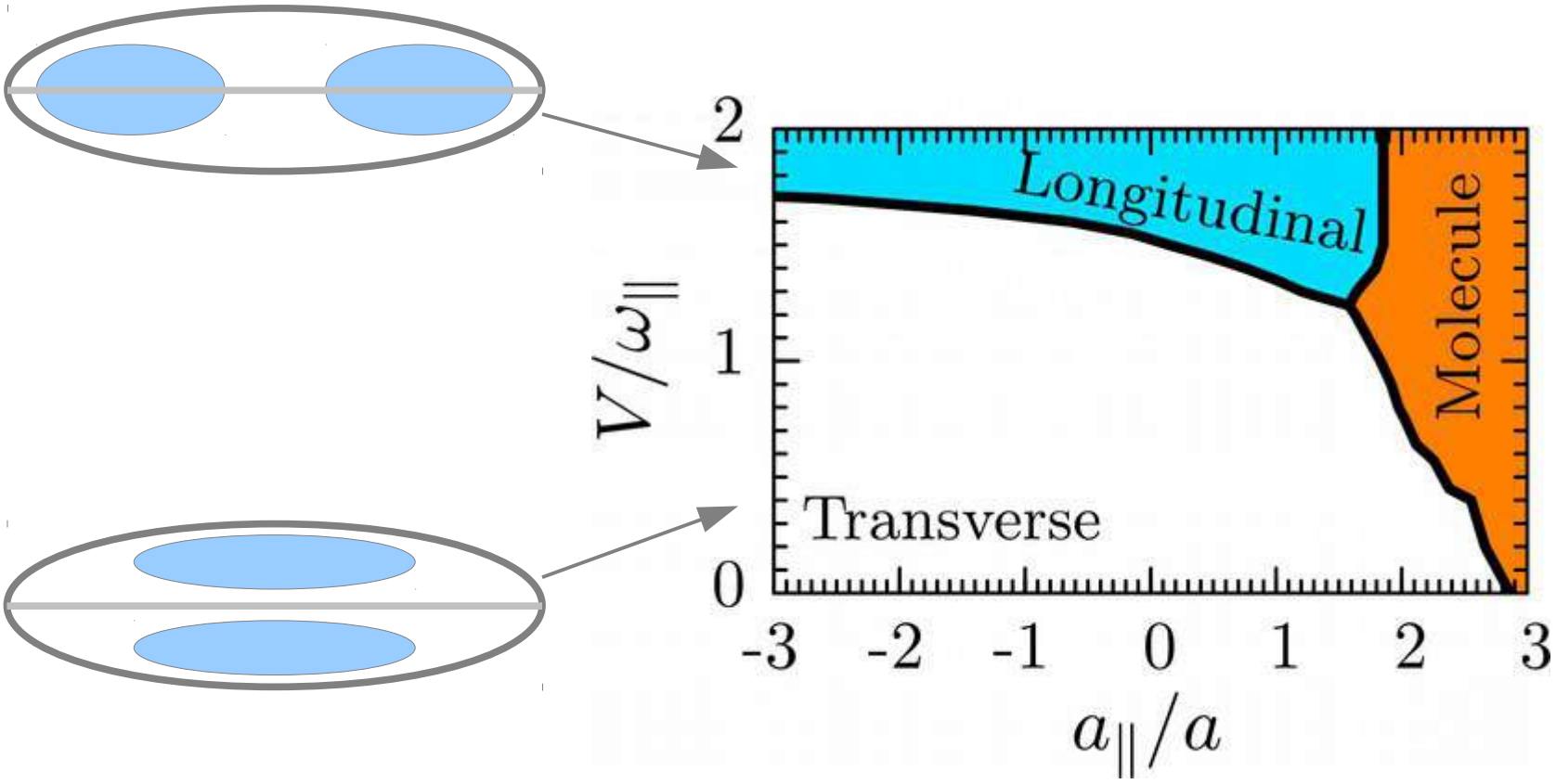
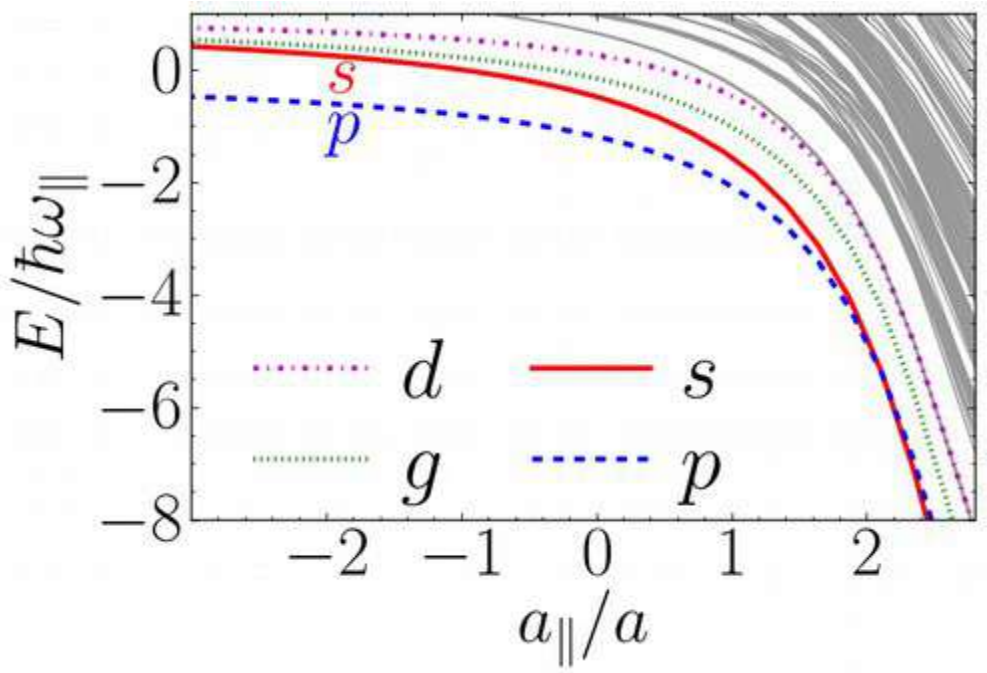




Inhomogeneous pairing



Inhomogeneous pairing



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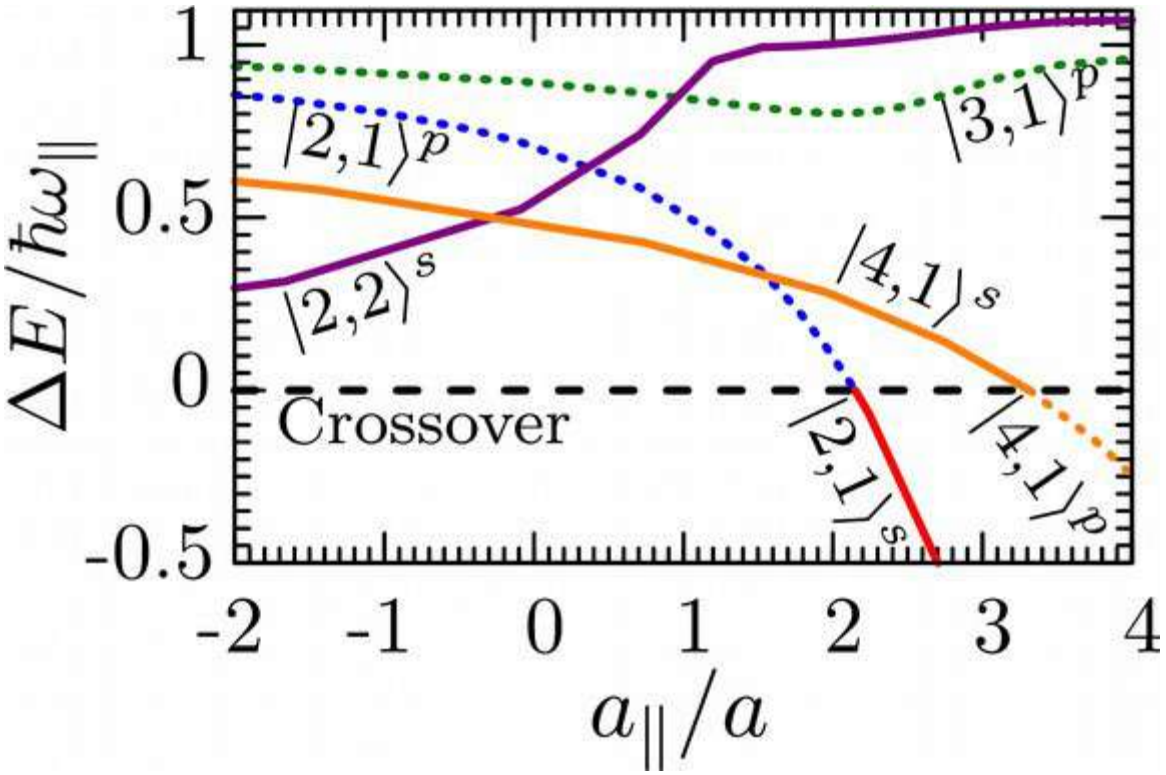
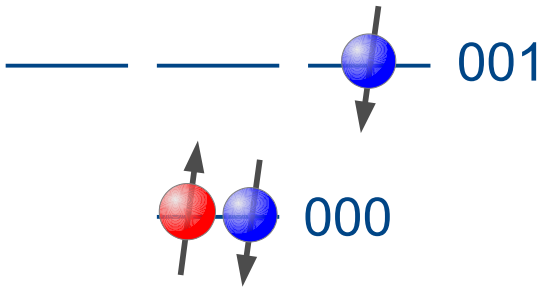
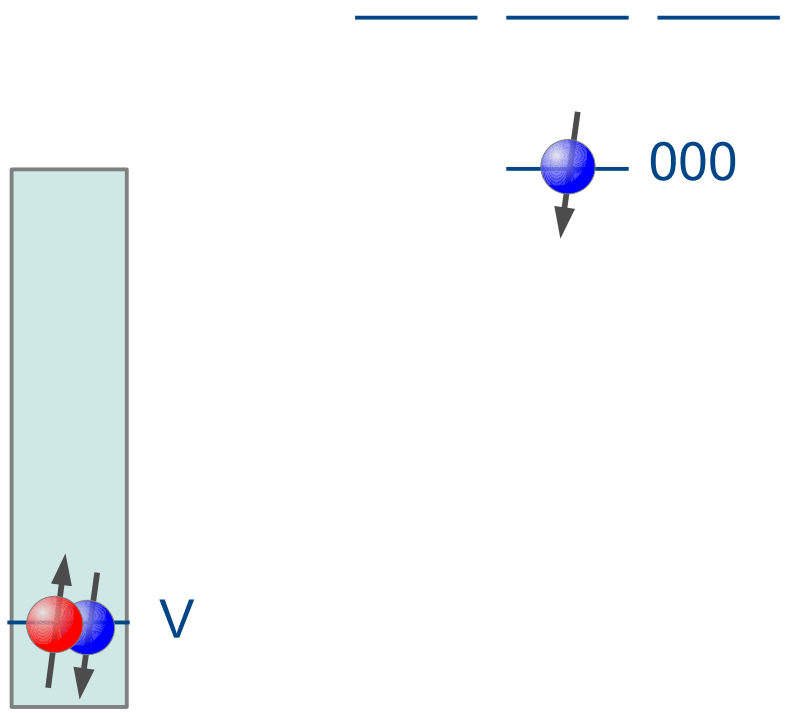


Inhomogeneous pairing

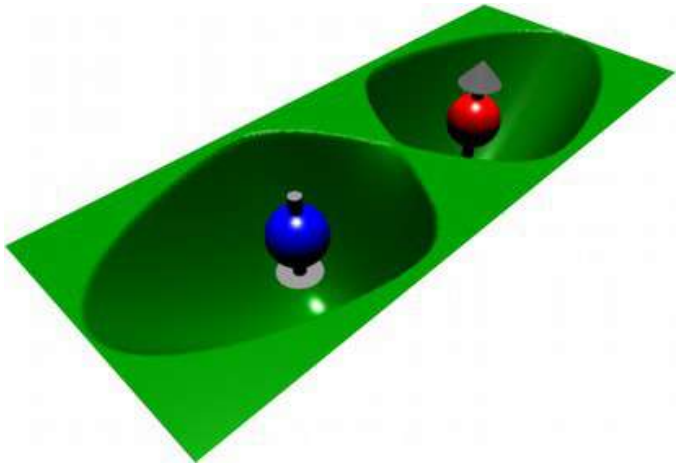
Weak interactions



Strong interactions

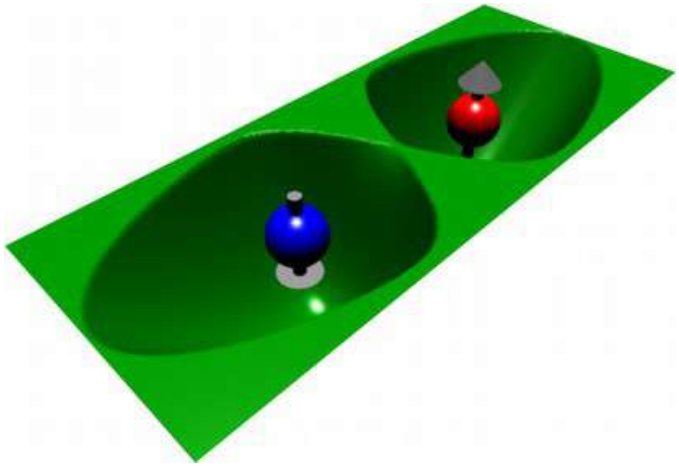


Inhomogeneous pairing



P.O. Bugnion & GJC, Phys. Rev. A **88**, 013601 (2013)

Inhomogeneous pairing



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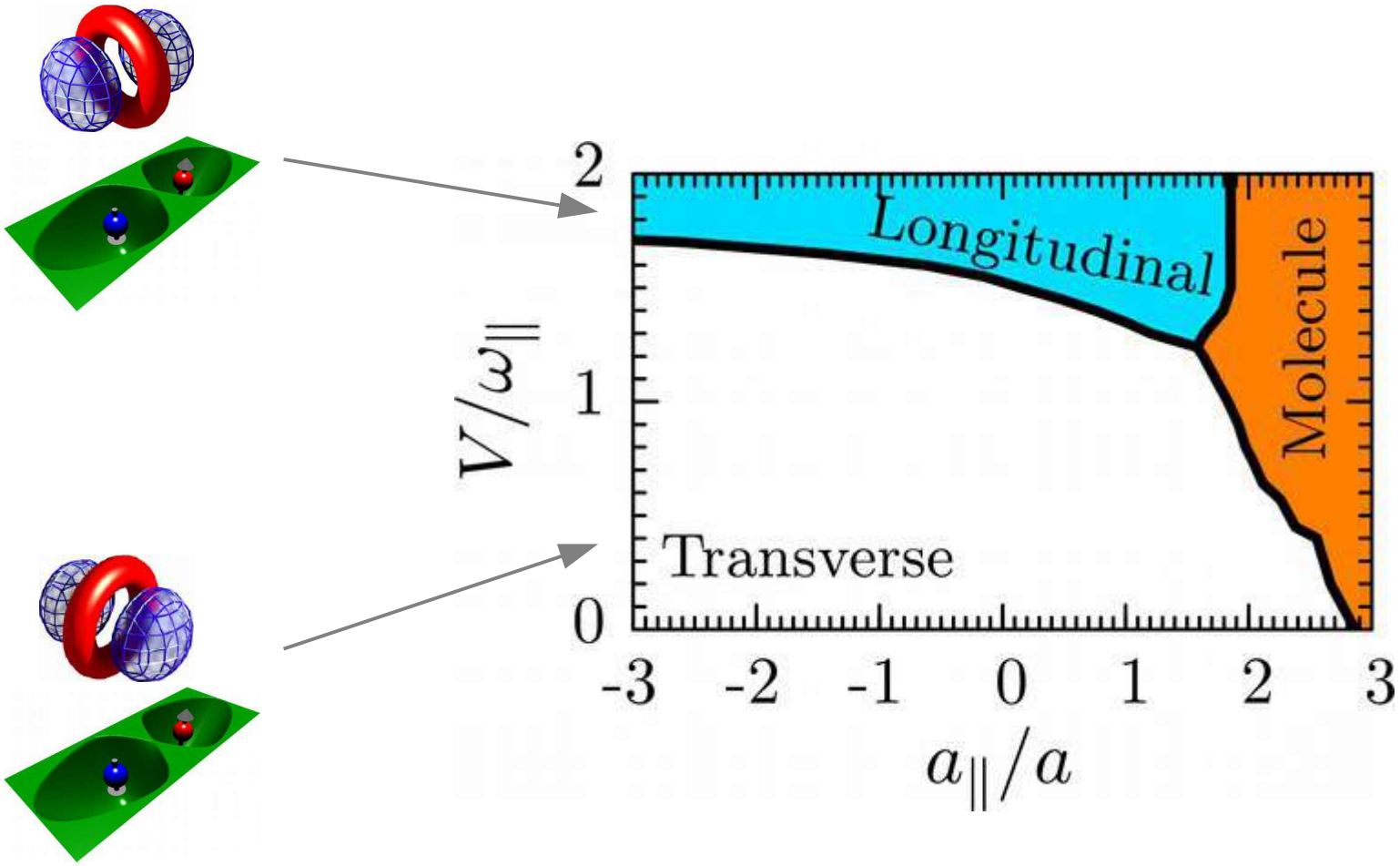
Inhomogeneous pairing