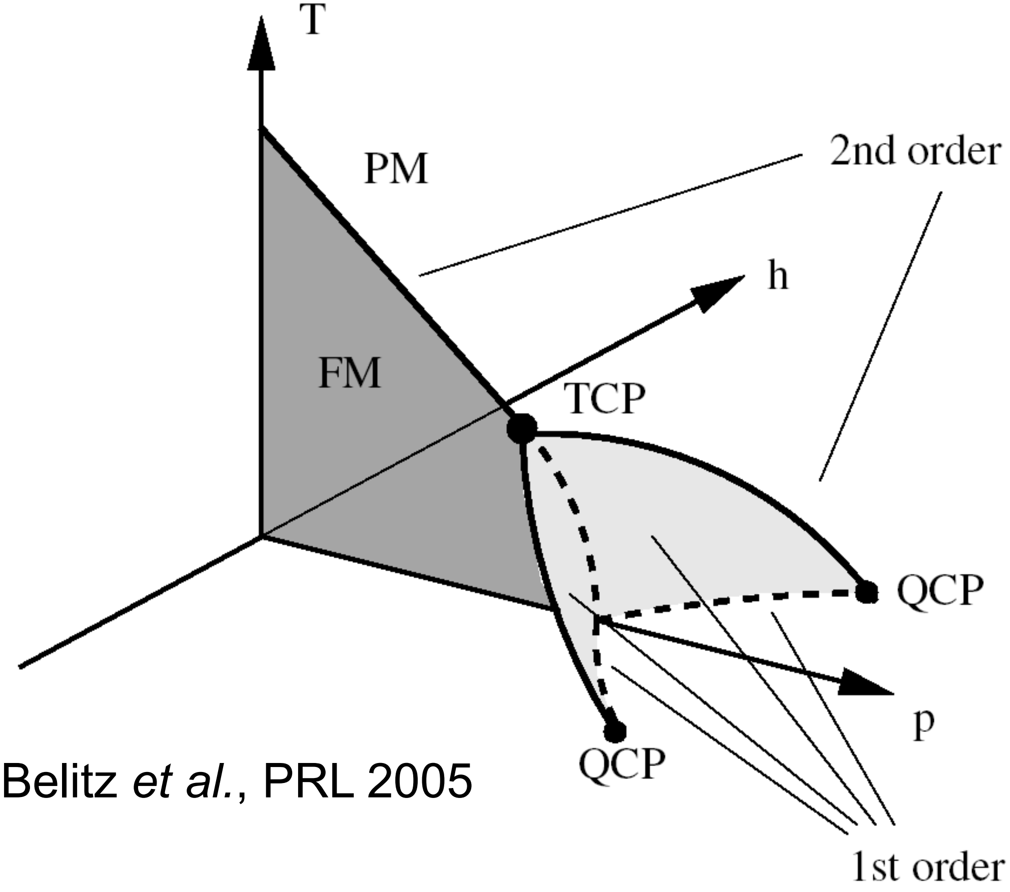
Quantum critical itinerant ferromagnetism



Belitz *et al.*, PRL 2005

Gareth Conduit

Cavendish Laboratory

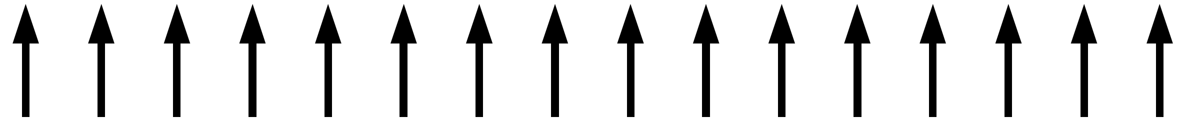
University of Cambridge

Two types of ferromagnetism

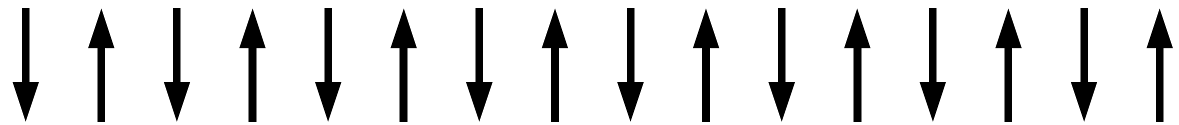
Gareth Conduit (Cavendish Laboratory)

- *Localized ferromagnetism*: moments localised in real space

Ferromagnet

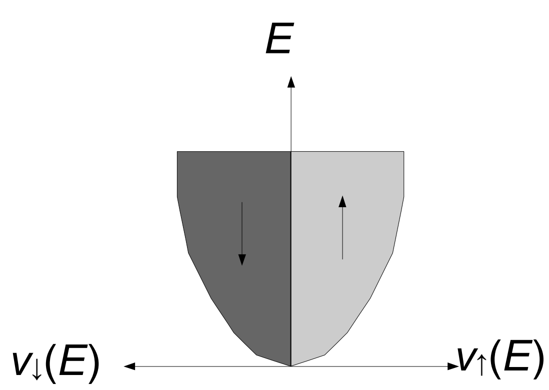


Antiferromagnet

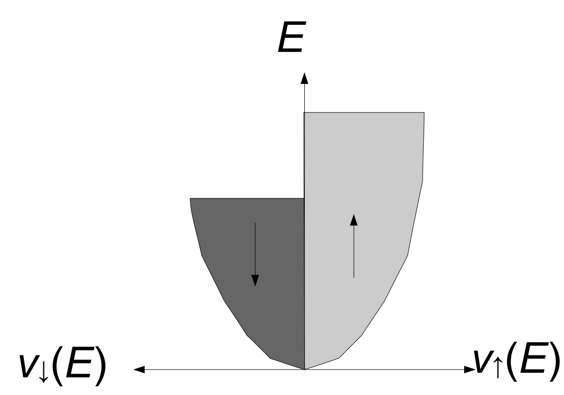


- *Itinerant ferromagnetism:* moments localised in reciprocal space

Not magnetised



Partially magnetised



Stoner model for itinerant ferromagnetism

Gareth Conduit (Cavendish Laboratory)

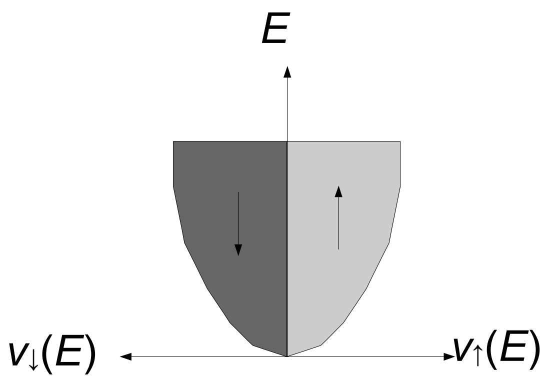
- Repulsive interaction energy $U=gn_\uparrow n_\downarrow$
- A ΔE shift in the Fermi surface causes:

  (i) Kinetic energy increase of $\frac{1}{2}\nu\Delta E^2$

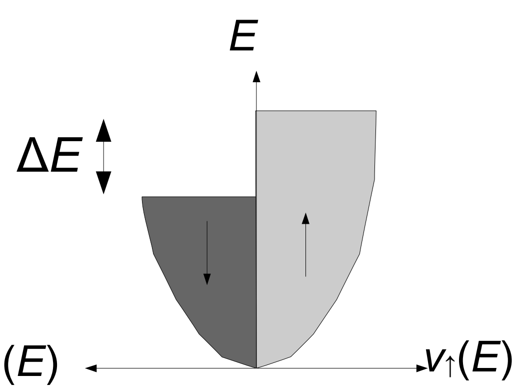
  (ii) Reduction of repulsion of $-\frac{1}{2}g\nu^2\Delta E^2$

- Total energy shift is $\frac{1}{2}\nu\Delta E^2(1-g\nu)$ so a ferromagnetic transition occurs if $g\nu>1$

Not magnetised



Partially magnetised



Ferromagnetism in iron

Gareth Conduit (Cavendish Laboratory)

- The Stoner model has a *second order* transition of e.g. iron and nickel

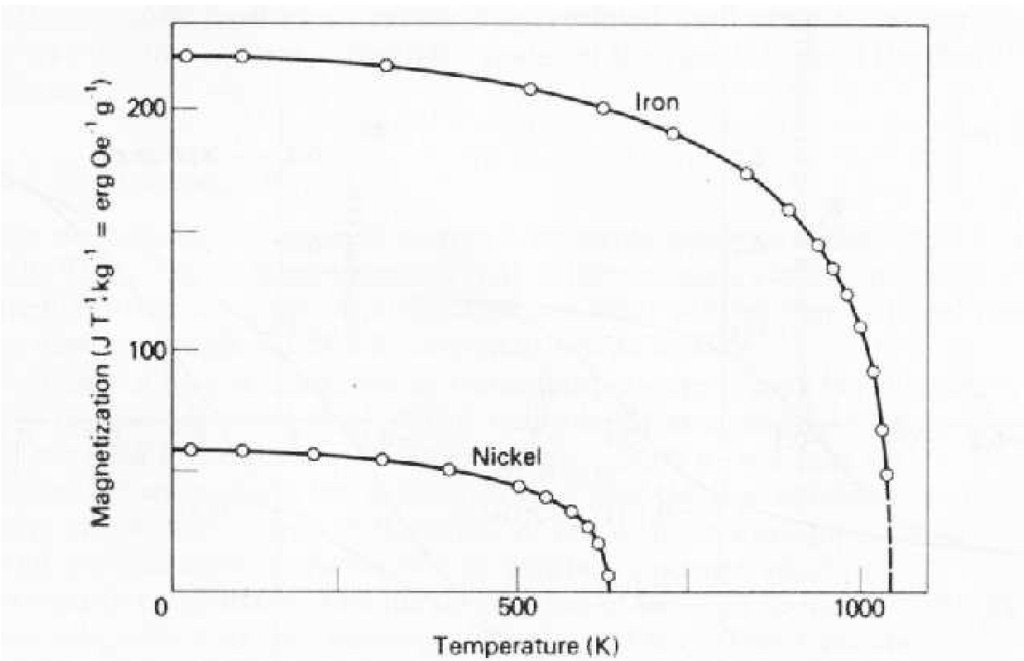


Figure 1.2 Spontaneous magnetization plotted against temperature for iron and nickel.

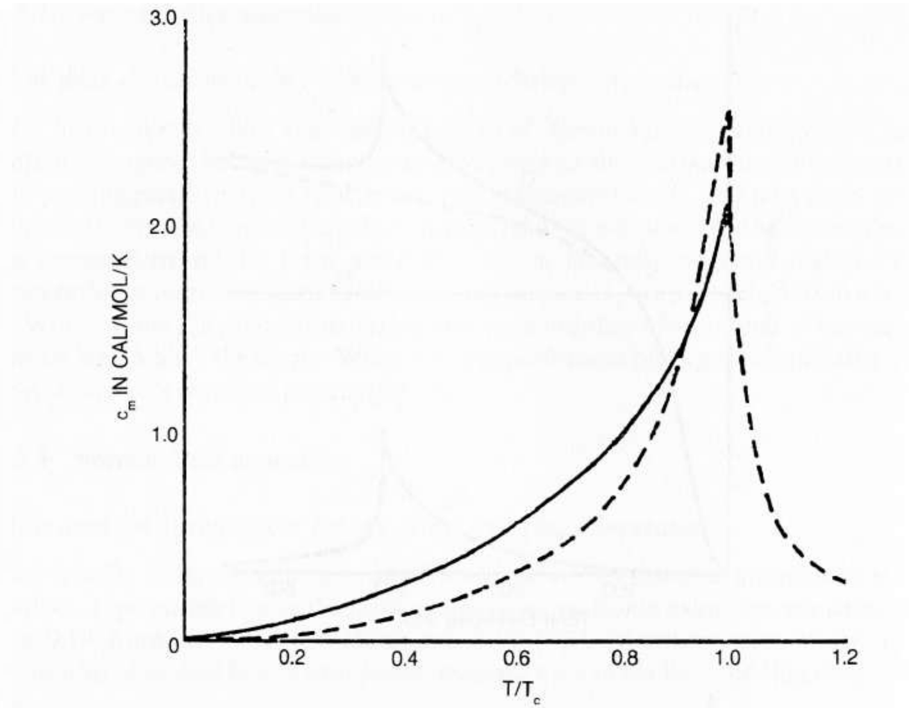


Fig. 9.20 Specific heat anomaly for nickel at its Curie point compared with the theoretical prediction.

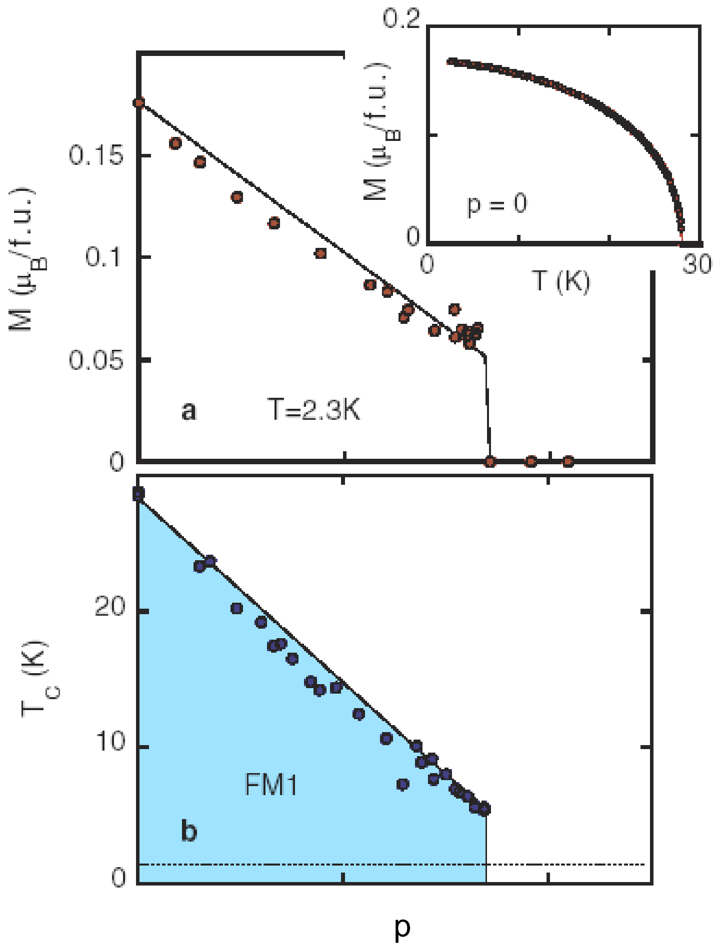
which is characterized by:

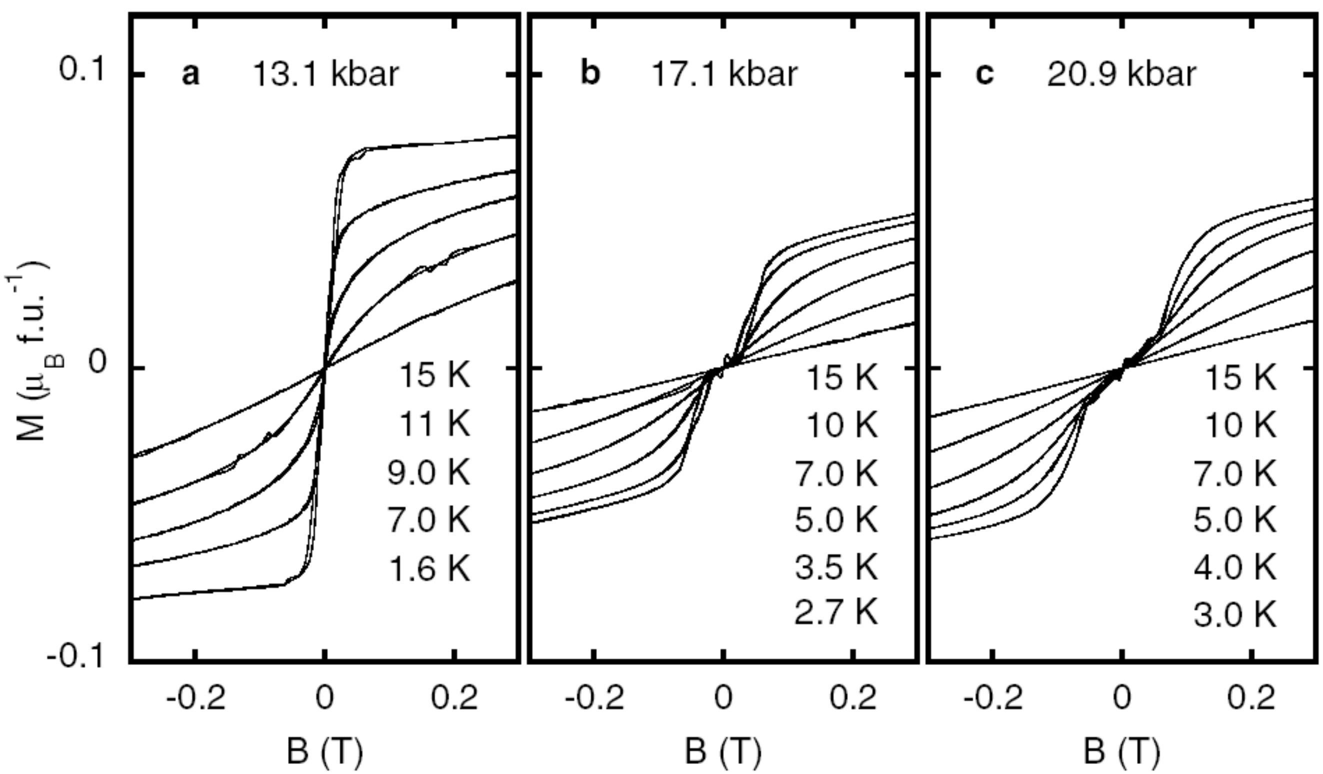
- Smoothly varying magnetisation
- A divergence of length-scales (peaked heat capacity and susceptibility)

Breakdown of Stoner criterion -- $ZrZn_2$

Gareth Conduit (Cavendish Laboratory)

- At low temperature and high pressure $ZrZn_2$ has a first order transition



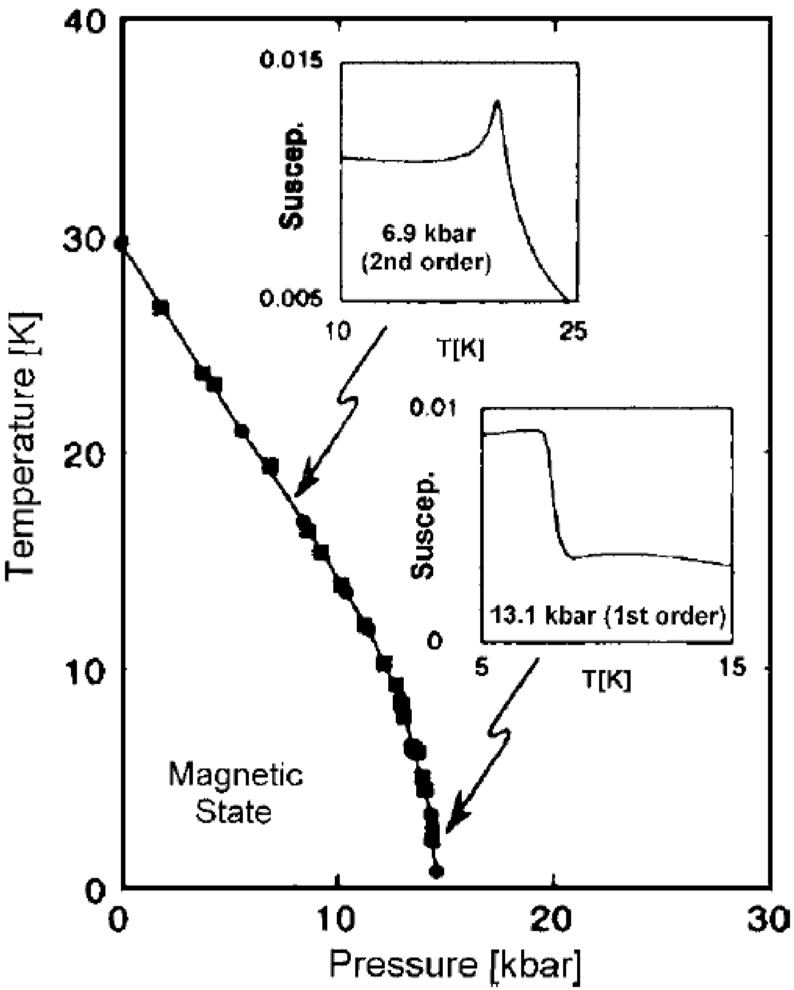


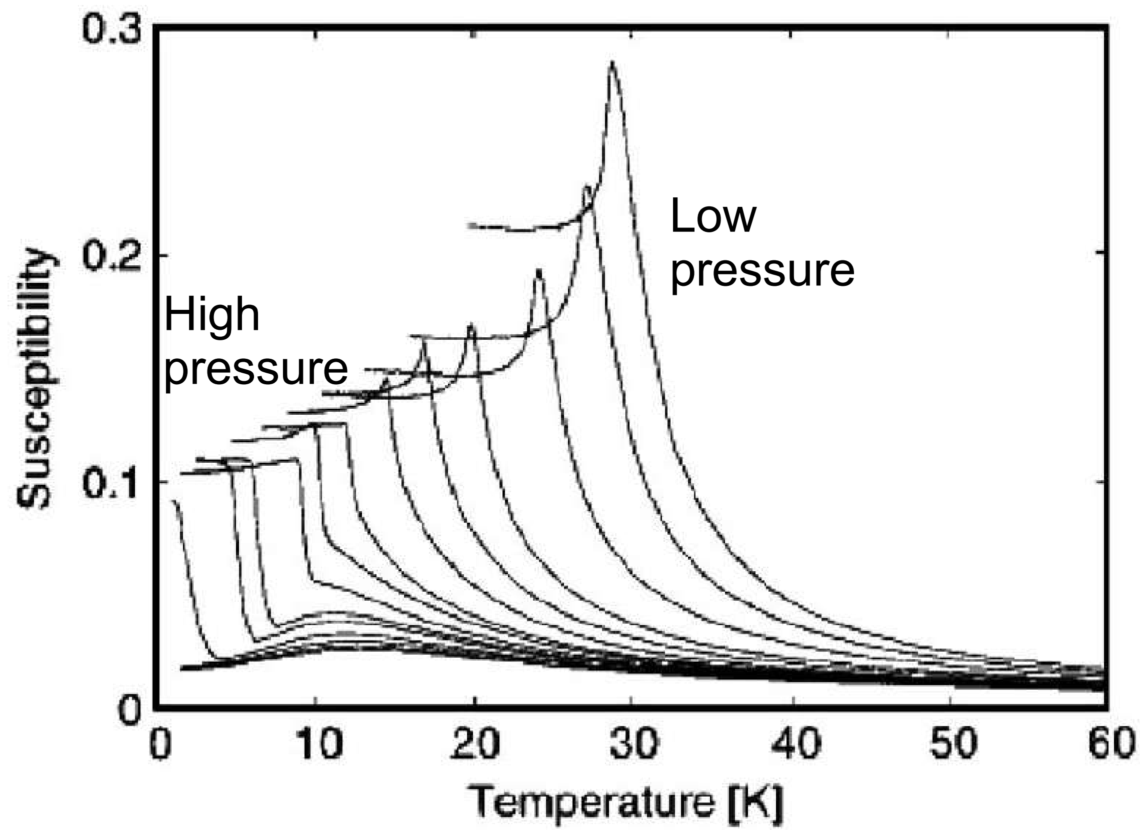
Uhlarz *et al.*, PRL 2004

Breakdown of Stoner criterion -- MnSi

Gareth Conduit (Cavendish Laboratory)

- MnSi also displays a first order phase transition





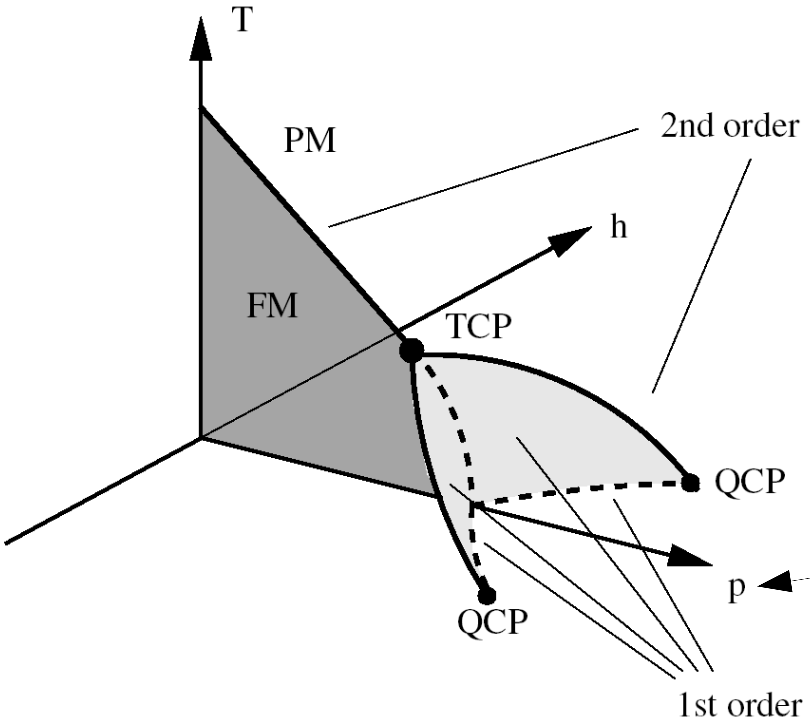
Pfleiderer *et al.*, PRB 1997

Pfleiderer *et al.*, PRB 1997
Vojta *et al.*, 1999 Ann. Phys. 1999

Breakdown of Stoner criterion

Gareth Conduit (Cavendish Laboratory)

- At low temperature UGe_2, $ZrZn_2$, MnSi, and others are first order



(interaction strength)

PM: Partially magnetised
FM: Fully magnetised
TCP: Tricritical point
QCP: Quantum critical point

- Here I describe two projects that investigate the first order behaviour:

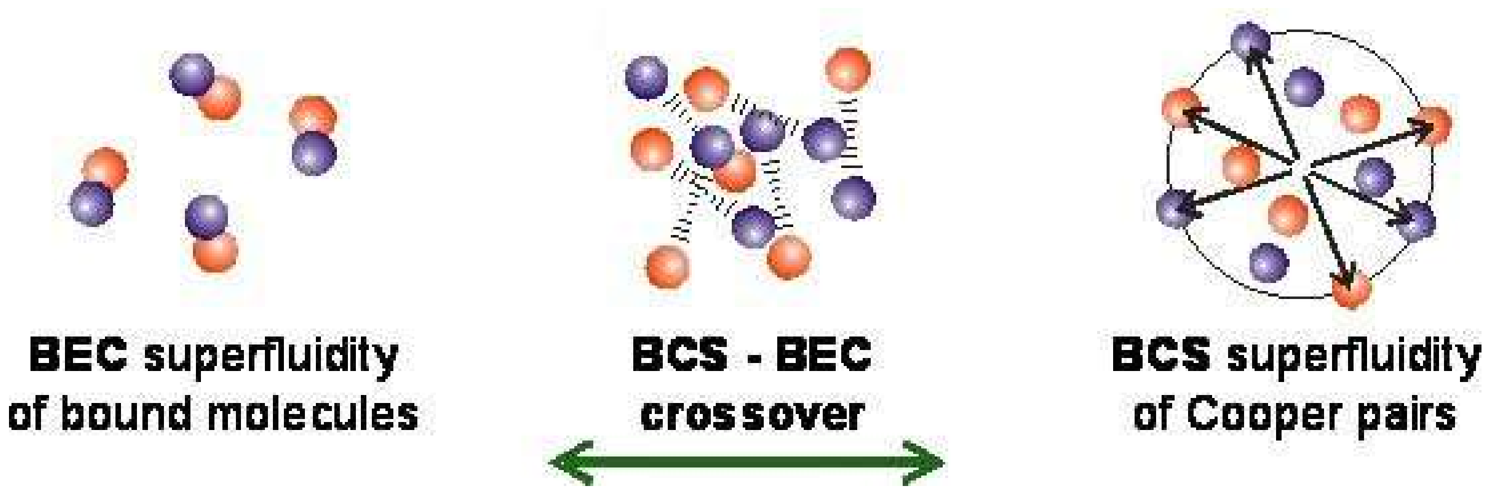
(i) Use atomic gases to probe the first order transition without the lattice

(ii) Motivated by the FFLO phase, apply the formalism to search for a putative textured phase

Cold atomic gases -- interactions

Gareth Conduit (Cavendish Laboratory)

- A gas of Fermionic atoms is prepared by laser and evaporative cooling to ~10^{-8}K
- Two-body contact collisions are controlled with a Feshbach resonance tuned by an external magnetic field
- Can tune from bound BEC molecules to weakly bound BCS regime[1]



Strong interactions Weak interactions

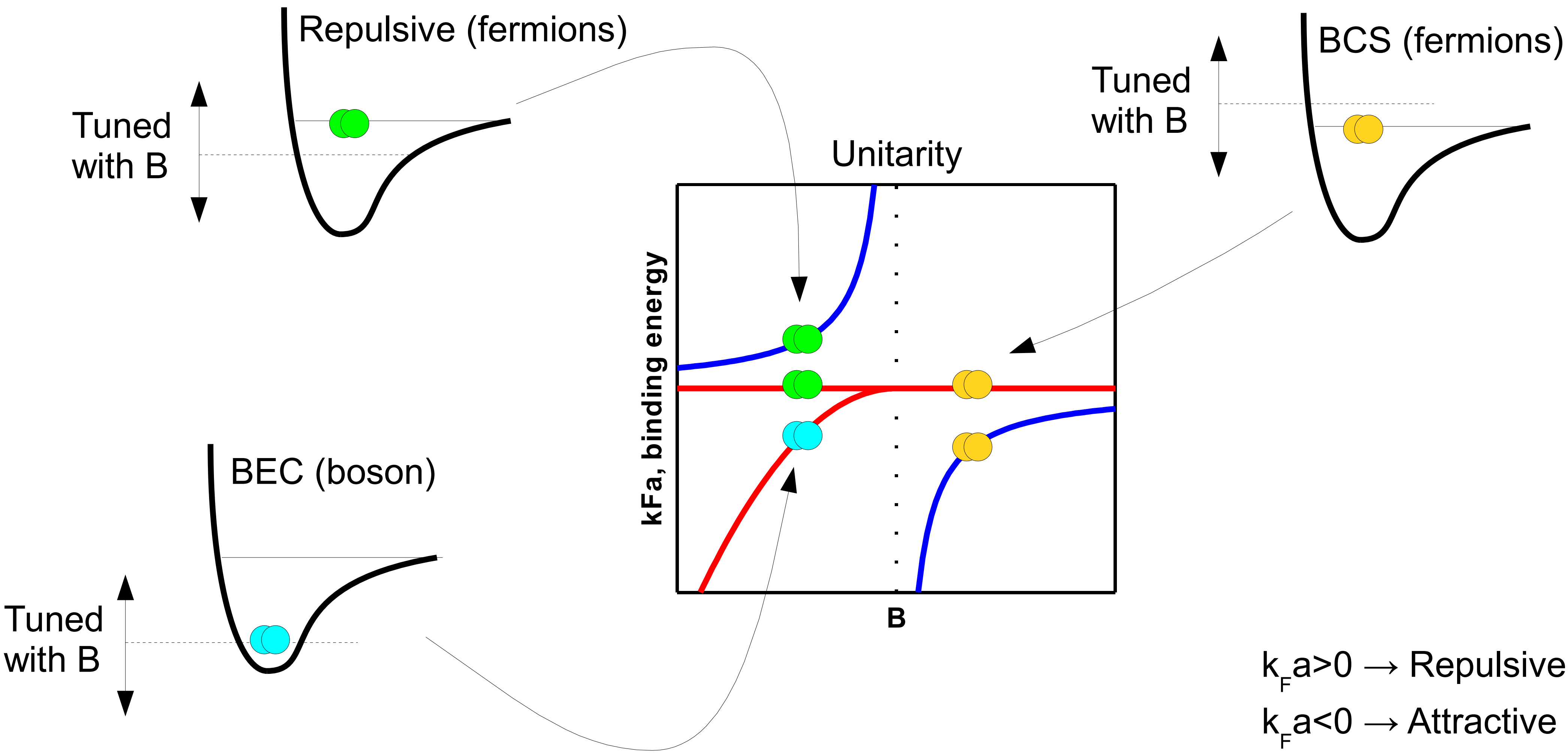
- Repulsive interactions might allow us to investigate itinerant ferromagnetism

[1]Lofus *et al*. PRL 2002, O'Hara *et al*. Science 2002, Bourdel *et al*. PRL 2003

Feshbach resonance

Gareth Conduit (Cavendish Laboratory)

- Control the relative energy level of states with an external magnetic field



$k_Fa>0 \rightarrow$ Repulsive

$k_Fa<0 \rightarrow$ Attractive

Cold atomic gases -- spin

Gareth Conduit (Cavendish Laboratory)

- Two fermionic atom species have a *pseudo-spin*:

^{40}K	$m_F=9/2$	maps to	spin 1/2
^{40}K	$m_F=7/2$	maps to	spin -1/2

- The up-and down spin particles *cannot* interchange -- population imbalance is fixed
- Atomic gases contain no disorder
- Atomic gases provide unprecedented levels of control allowing investigators to probe solid state phenomena e.g. Hubbard model[1], superfluid vortices[2], Josephson effects[3], FFLO[4], and Kosterlitz-Thouless phase transition[5]

[1]Greiner *et al*. Nature 2002, [2]Abo-Shaeer *et al*. Science 2001, [3]Albiez *et al*. PRL 2005, [4]GJC *et al*. PRB 2008, [5]Hadzibabic *et al*. Nature 2006

Population imbalance ferromagnetism

Gareth Conduit (Cavendish Laboratory)

- A spin up and a spin down particle (S_z=0) in triplet and singlet states:

$\|\uparrow\uparrow\rangle$	S=1, S_z=1	State not possible as S_z has changed
$\|\downarrow\downarrow\rangle$	S=1, S_z=-1	State not possible as S_z has changed
$(\|\uparrow\downarrow\rangle + \|\downarrow\uparrow\rangle)/\sqrt{2}$	S=1, S_z=0	Magnetic moment in plane
$(\|\uparrow\downarrow\rangle - \|\downarrow\uparrow\rangle)/\sqrt{2}$	S=0, S_z=0	Non-magnetic state

- Ferromagnetism, if favourable, must form in plane

Analytical method

Gareth Conduit (Cavendish Laboratory)

- System free energy $F=-k_B T \ln Z$ is found via the partition function

$$Z=\sum_{\{m(x,t),n(x,t)\}} \exp(-E[m(x,t),n(x,t)]/k_B T)$$

the summation includes spatial and temporal fluctuations of magnetisation and density

- Using only the average magnetisation and density:

$$m(x,t)=\bar{m}$$

$$n(x,t)=\bar{n}$$

gives

$$F \propto (1-g\nu)\bar{m}^2$$

i.e. the Stoner criterion

Method of steepest descent

Gareth Conduit (Cavendish Laboratory)

- Suppose the partition function takes the form

$$Z=\int_0^\infty \exp(-m+s\ln m)\,dm$$

- Expand about the maximum of the function at $m=s$:

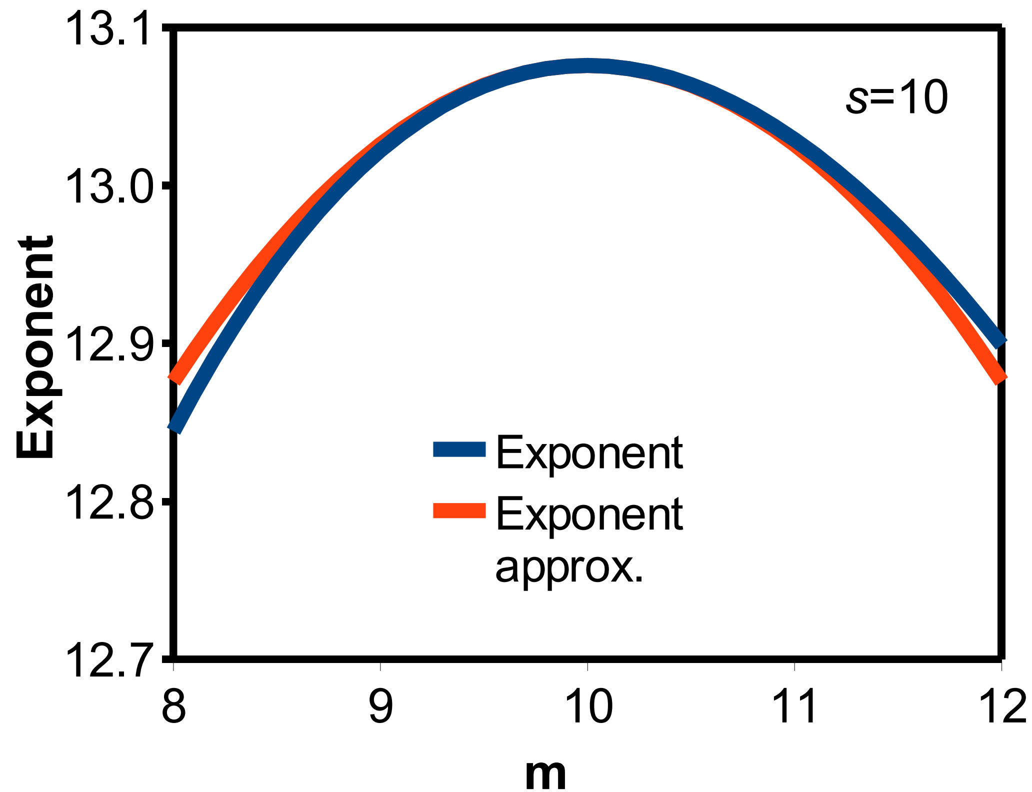
$$Z=\int_0^\infty \exp(-s(1-\ln s)-(m-s)^2/2s+O(m-s)^3)\,dm$$

- Following the Gaussian integral

$$Z\approx\sqrt{2\pi s}\exp(-s(1-\ln s))$$

- This is Stirling's formula

$$s!\approx\sqrt{2\pi s}\,s^s e^{-s}$$



Consequences of fluctuations

Gareth Conduit (Cavendish Laboratory)

- In a similar way we can expand the energy in magnetisation to second order to account for fluctuations

$$Z=\sum_{\{m(x,t),n(x,t)\}}\exp(-E[m,n]/k_BT)$$
$$=\sum_{\{\delta m(x,t),\delta n(x,t)\}}\exp\left(\frac{-1}{k_BT}\left(E[\bar{m},\bar{n}]+(\delta m\quad \delta n)\begin{pmatrix}E^{(2,0)} & E^{(1,1)}\\ E^{(1,1)} & E^{(0,2)}\end{pmatrix}\begin{pmatrix}\delta m\\ \delta n\end{pmatrix}\right)\right)$$

- Larkin & Pikin [Zh. Eksp. Teor. Fiz. 1969] included auxiliary fluctuations of the lattice which introduced a negative magnetisation term, driving the transition first order

$$rm^2+um^4+a\phi^2\pm 2am^2\phi=rm^2+(u-a)m^4+a(\phi\pm m^2)^2=rm^2+(u-a)m^4$$

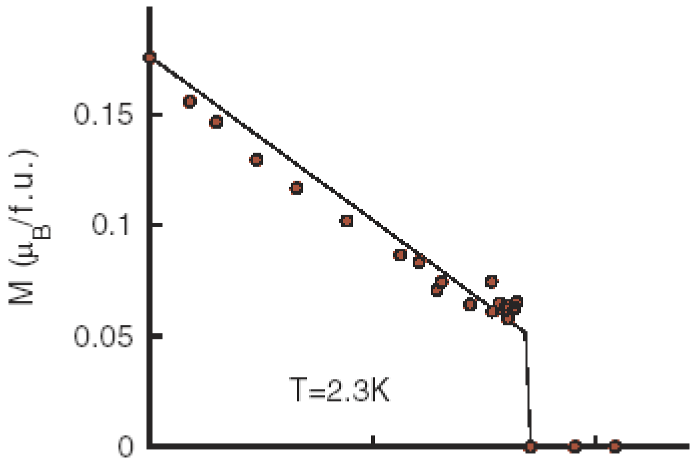
- Previous work on itinerant ferromagnetism considered a mean field Ginzburg-Landau expansion[1] or non-analyticities to examine the transition[2]

[1]Belitz *et al*. PRL 2005, [2]Belitz *et al*. PRL 2002

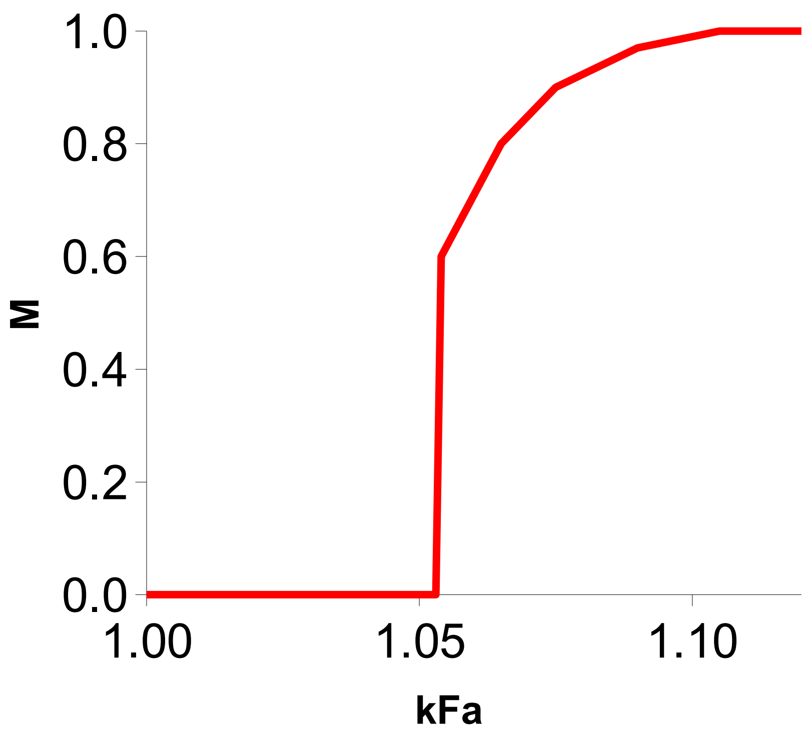
Fluctuation corrections

Gareth Conduit (Cavendish Laboratory)

- We include corrections due to dynamic quantum fluctuations in magnetisation in x, y, and z directions, and also account for fluctuations in density
- Similarly here considering the soft transverse magnetic fluctuations drives the transition of the longitudinal first order
- The results give the following phase diagram



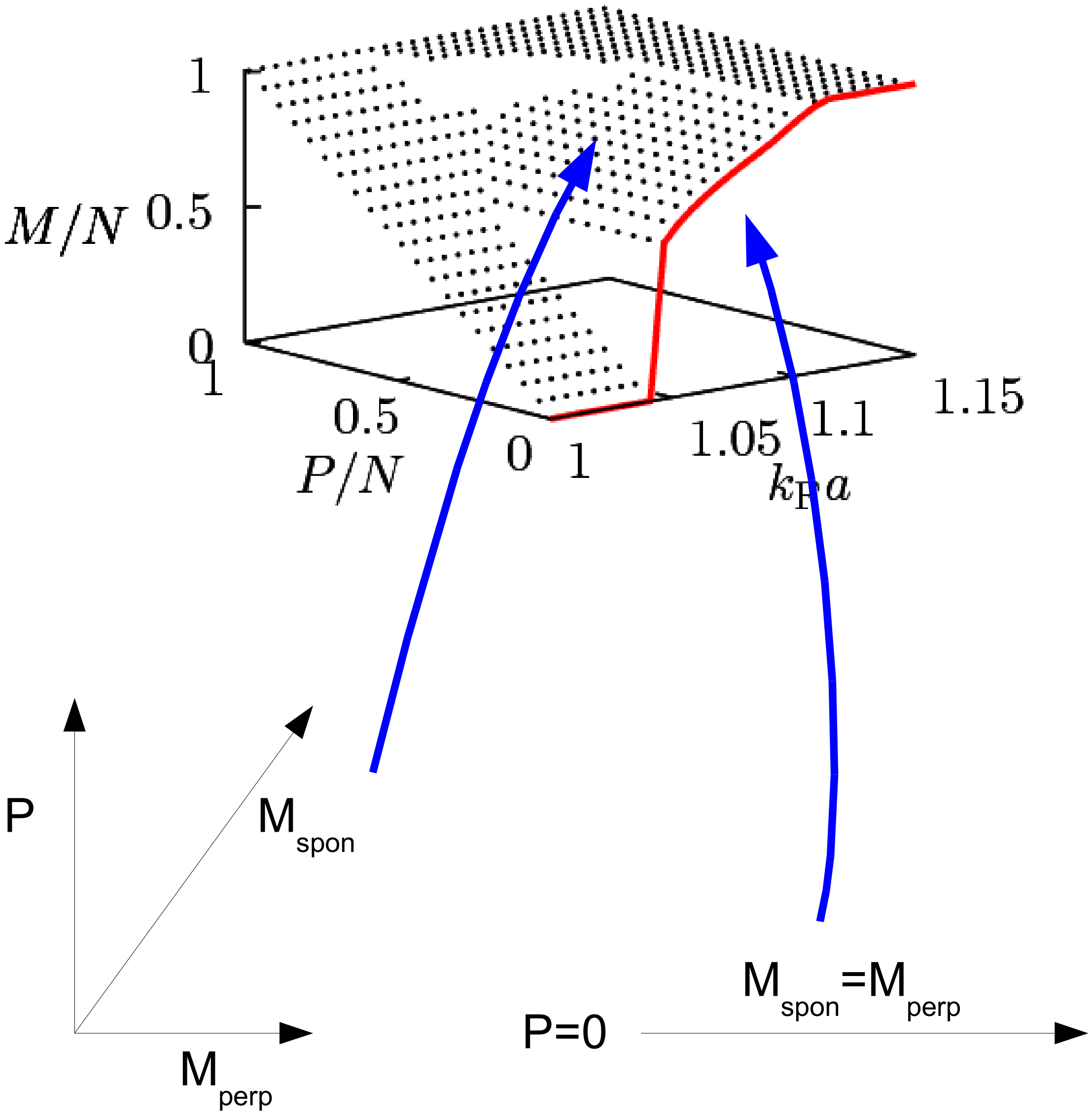
Uhlarz *et al.*, PRL 2004



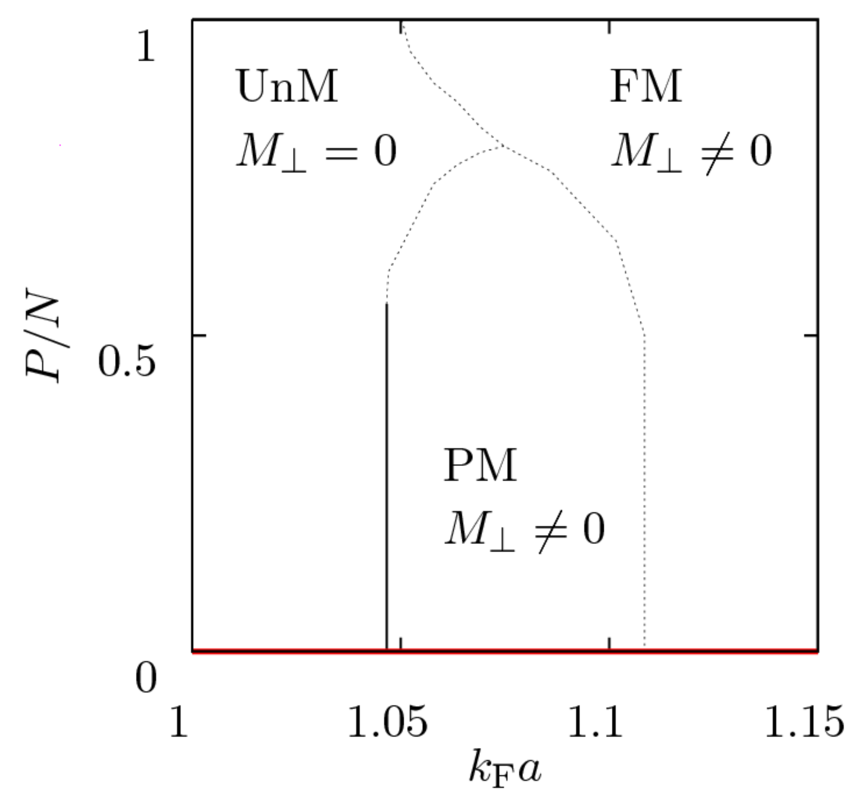
Population imbalanced case

Gareth Conduit (Cavendish Laboratory)

- With population imbalance *P* in the canonical regime we obtain



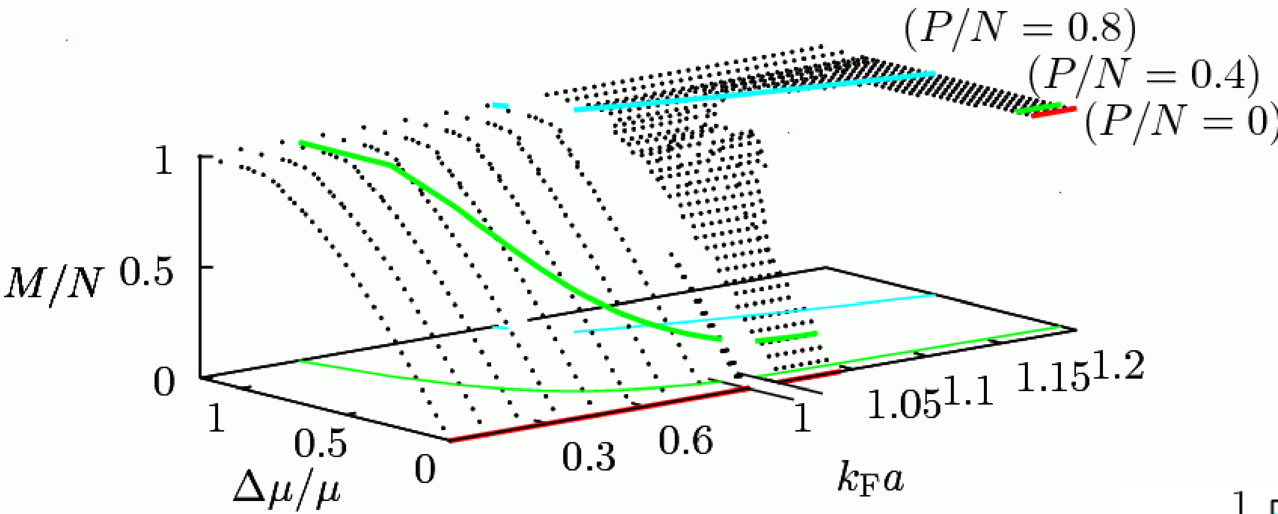
UnM: Unmagnetised
PM: Partially magnetised
FM: Fully magnetised

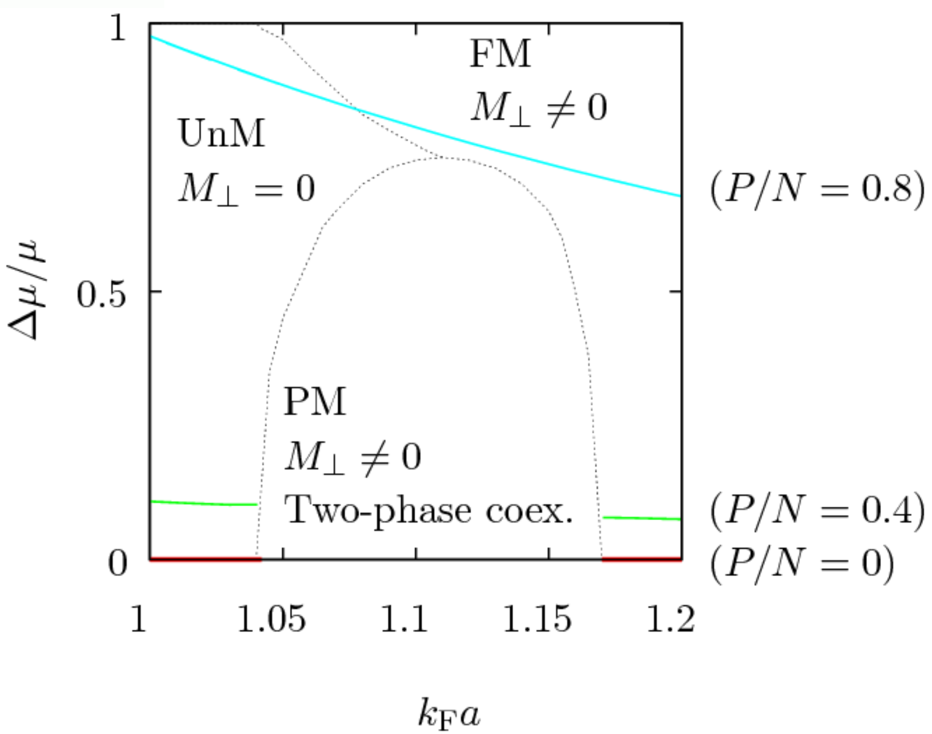


Grand canonical ensemble

Gareth Conduit (Cavendish Laboratory)

- In the grand canonical ensemble we obtain

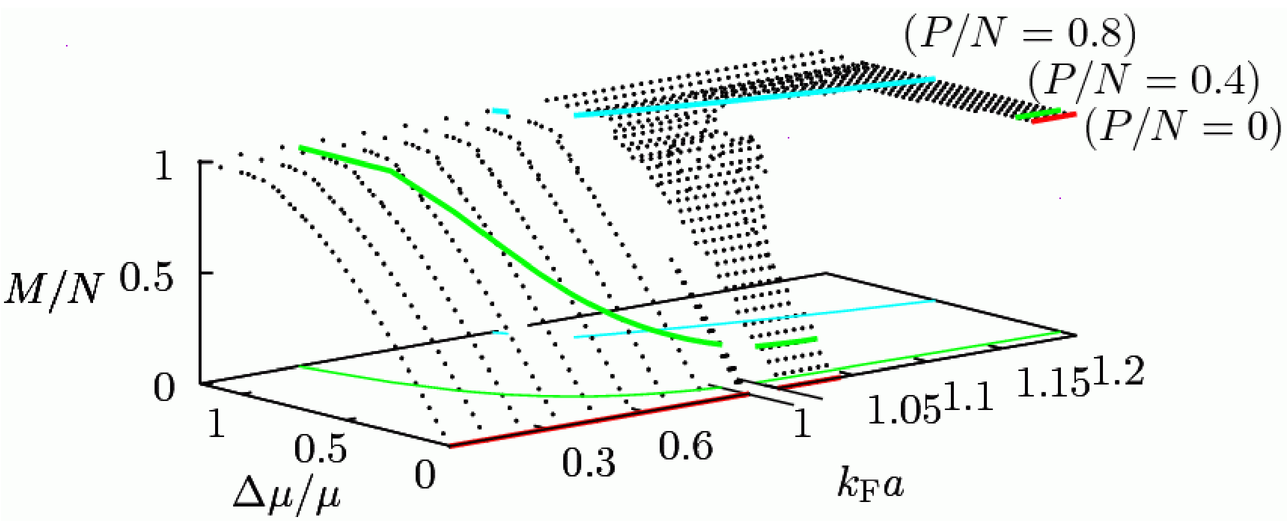


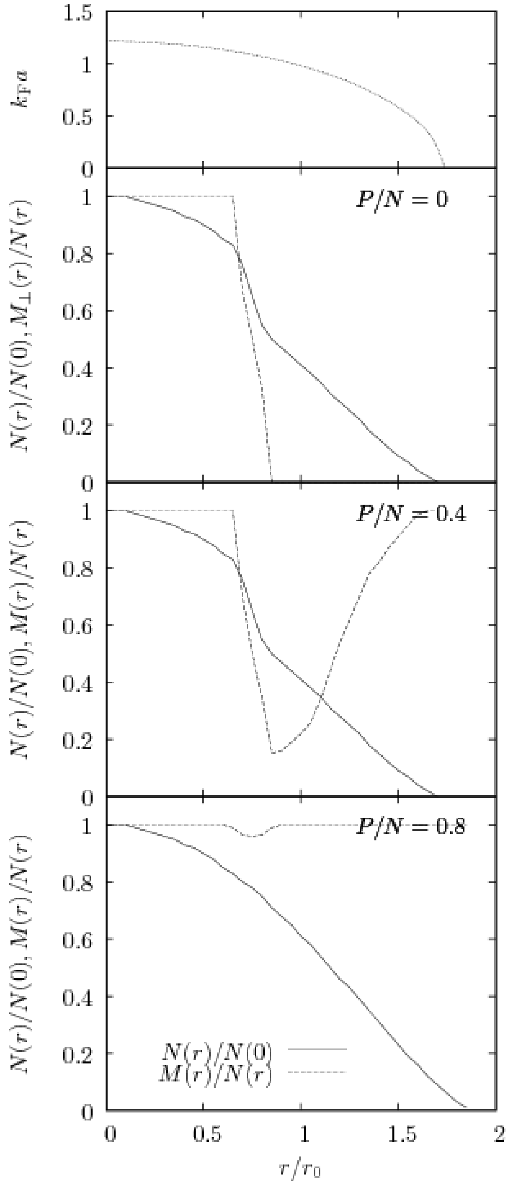


Trap behaviour

Gareth Conduit (Cavendish Laboratory)

- Trap behaviour corresponds to three trajectories in the phase diagram

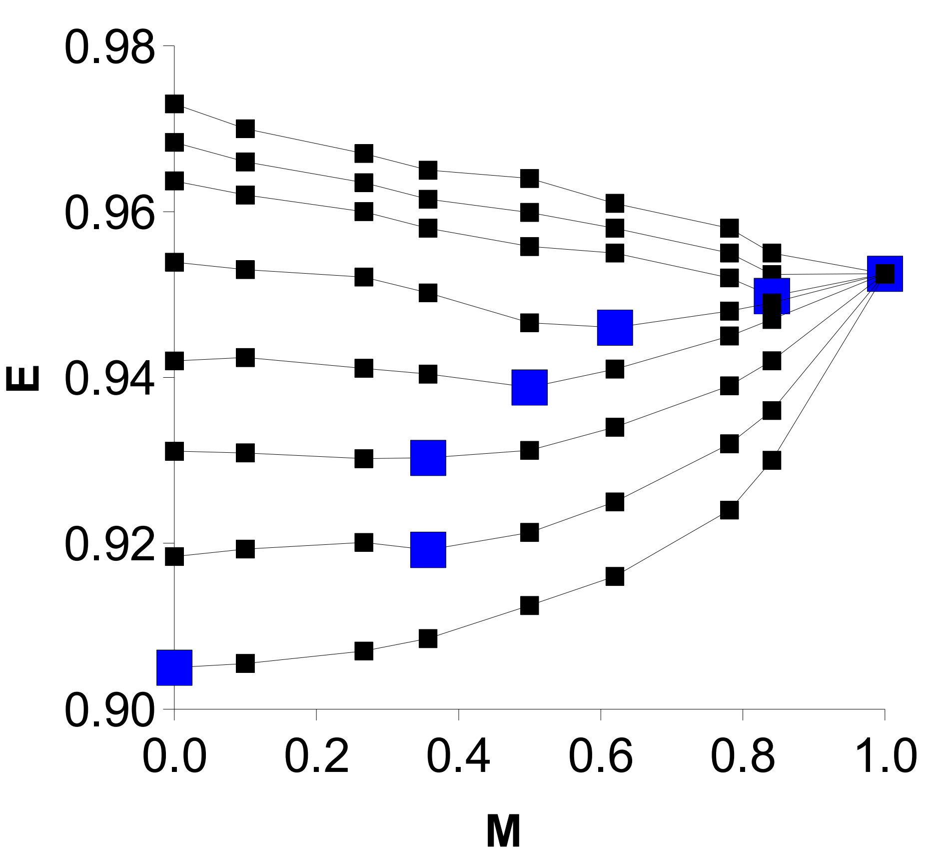


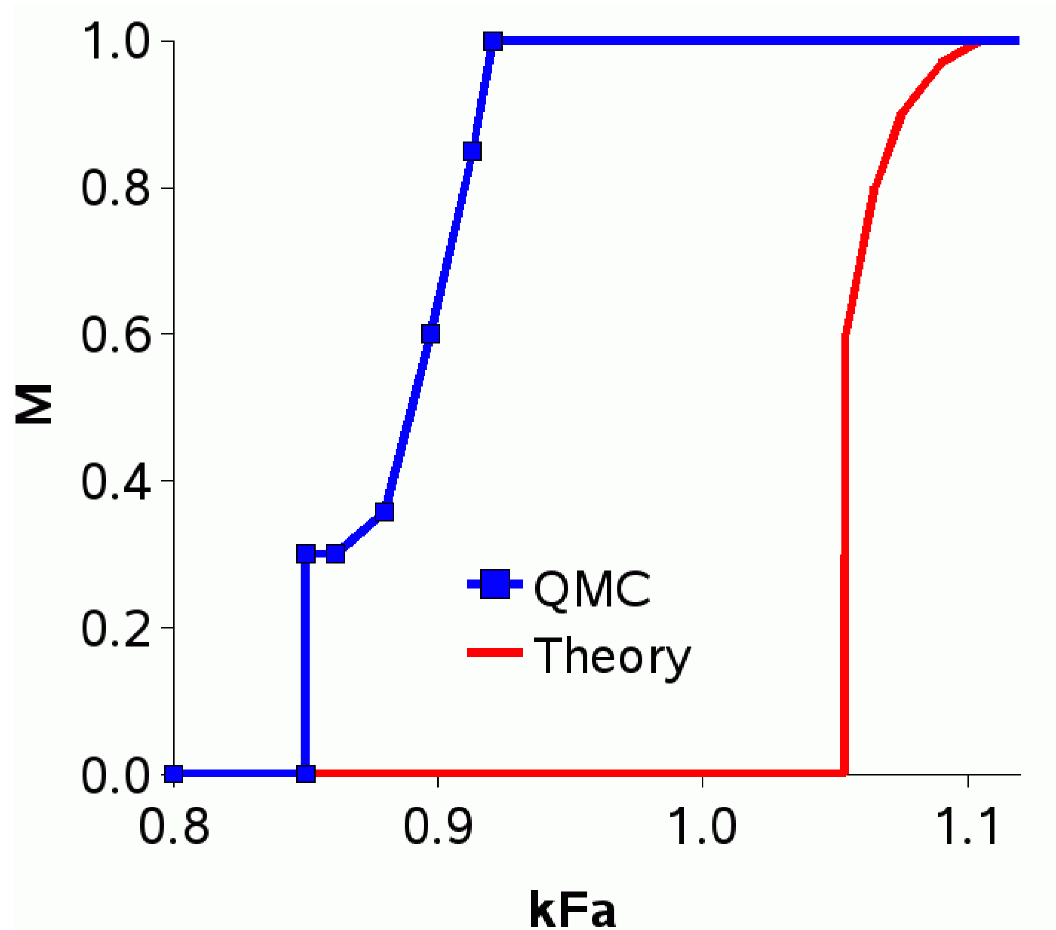


QMC calculations

Gareth Conduit (Cavendish Laboratory)

- Fluctuation corrections are not exact and higher order terms might destroy the first order phase transition
- Exact (except for the fixed node approximation) Quantum Monte Carlo calculations confirmed a first order phase transition



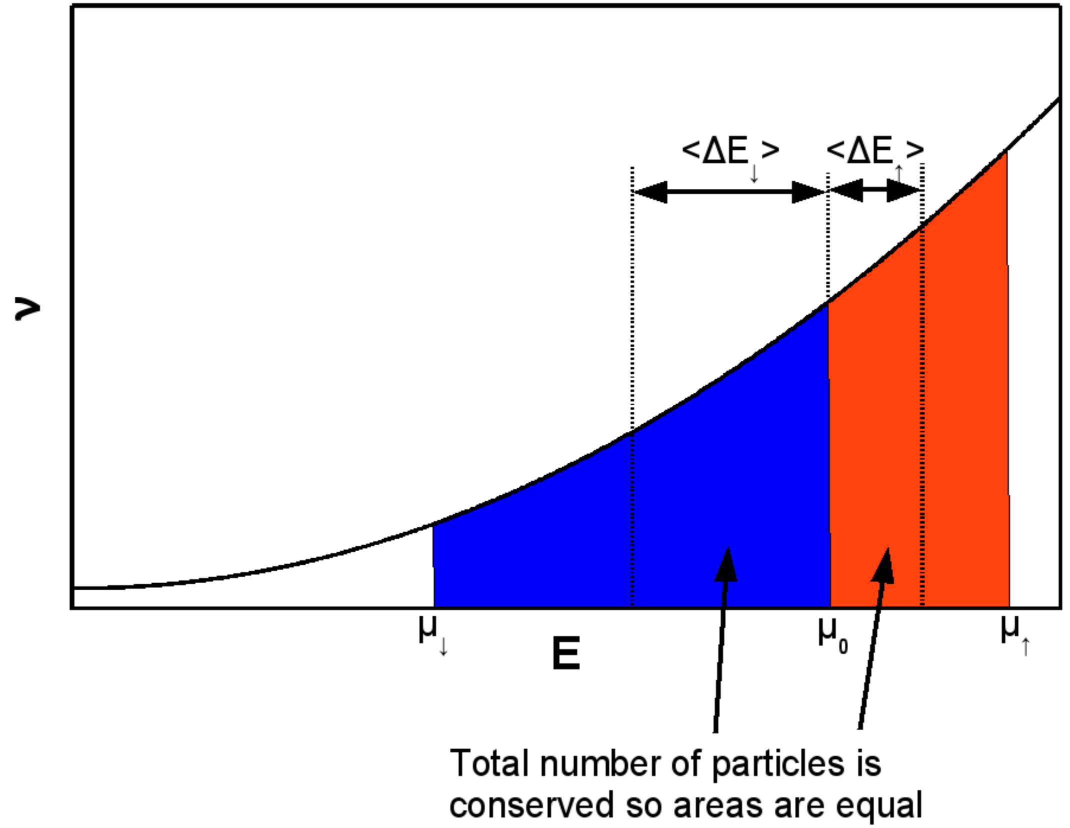


Wohlfarth Rhodes criterion

Gareth Conduit (Cavendish Laboratory)

- Do fluctuations influence the transition through the density of states?
- The first order transition could be caused by a peak in the density of states [Sandeman *et al*. PRL 2003, Pfleiderer *et al*. PRL 2002]
- If the density of states ν(E) changes rapidly with energy then a ferromagnetic transition is favourable when [Binz *et al*. EPL 2004]

$$\nu \nu'' > 3 (\nu')^2$$



Improved Wohlfarth Rhodes criterion

Gareth Conduit (Cavendish Laboratory)

- Accounting for changes in the energy spectrum ε gives criterion

$$\int_0^u \varepsilon^{(0,4)}(w,0)\mathrm{d}w + 4\varepsilon^{(0,3)}(u,0) + 6\varepsilon^{(1,2)}(u,0) + 4\varepsilon^{(2,1)}(u,0) + \varepsilon^{(3,0)}(u,0) < 0$$

Overall change in energy spectrum during the transition

How energy spectrum changes during transition at the Fermi surface

Wohlfarth Rhodes criterion

Differential of energy spectrum curve → $\varepsilon^{(a,b)}$ ← Differentiate energy spectrum wrt changing Fermi surface

- The terms have magnitude

Term	Expansion
$\int_0^u \varepsilon^{(0,4)}(w,0)\mathrm{d}w$	$0.0k_\mathrm{F}a + 0.0086(k_\mathrm{F}a)^2$
$4\varepsilon^{(0,3)}(u,0)$	$0.0k_\mathrm{F}a - 0.04(k_\mathrm{F}a)^2$
$6\varepsilon^{(1,2)}(u,0)$	$0.024(k_\mathrm{F}a)^2$
$4\varepsilon^{(2,1)}(u,0)$	$0.0(k_\mathrm{F}a)^2$
$\varepsilon^{(3,0)}(u,0)$	$2^{-3/2}/27 - 0.0055(k_\mathrm{F}a)^2$

Transition due to changing energy spectrum at the Fermi surface

Summary of uniform work

Gareth Conduit (Cavendish Laboratory)

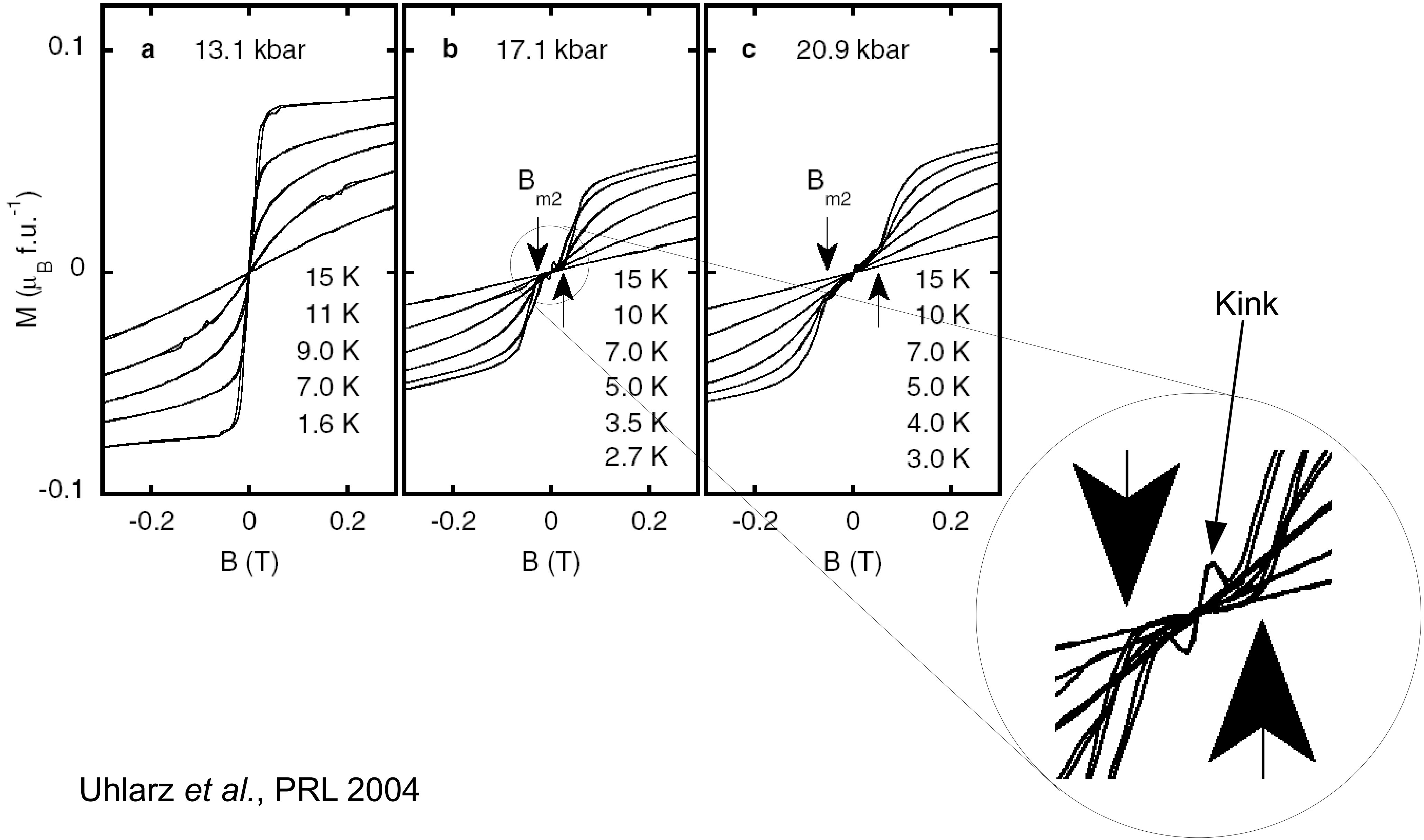
- Consideration of corrections due to fluctuations in magnetisation and density revealed a first order phase transition
- Nature of transition confirmed by Quantum Monte Carlo calculations
- Shed light on relation to features in the density of states

- Motivated by FFLO and experiment now examine a putative textured ferromagnetic phase

$ZrZn_2$

Gareth Conduit (Cavendish Laboratory)

- Kink in magnetisation indicative of metamagnetic phase

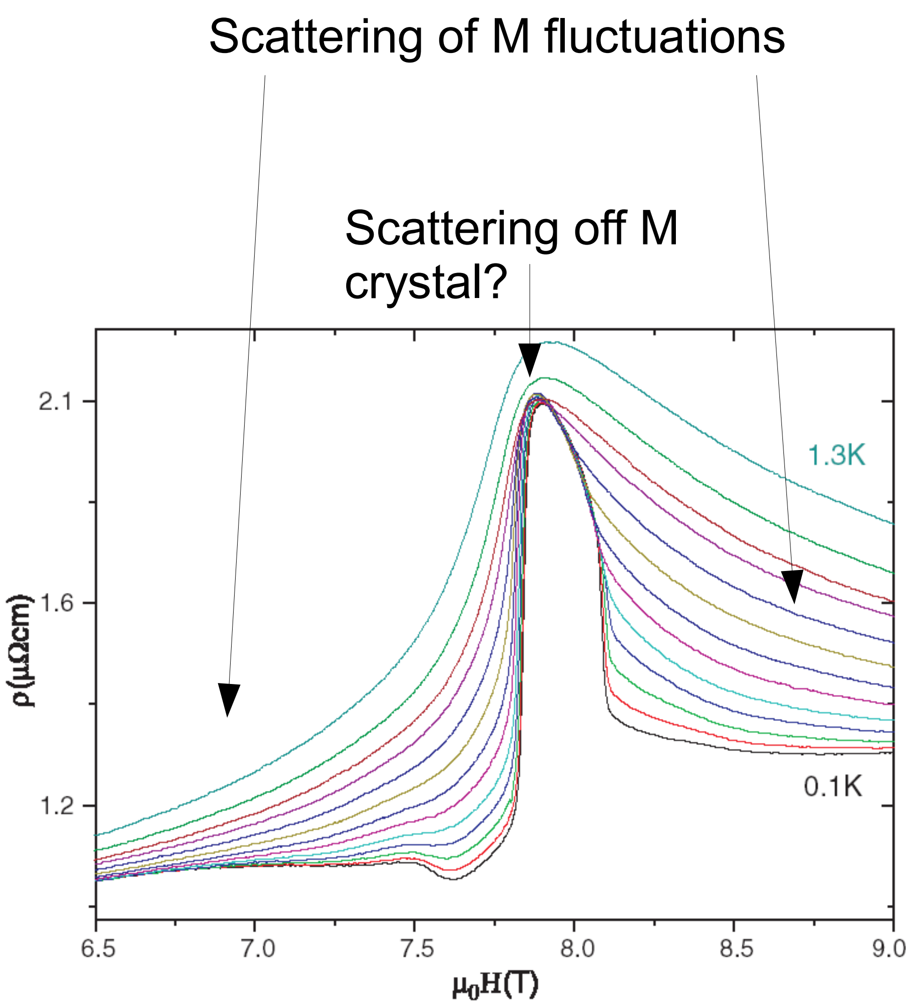


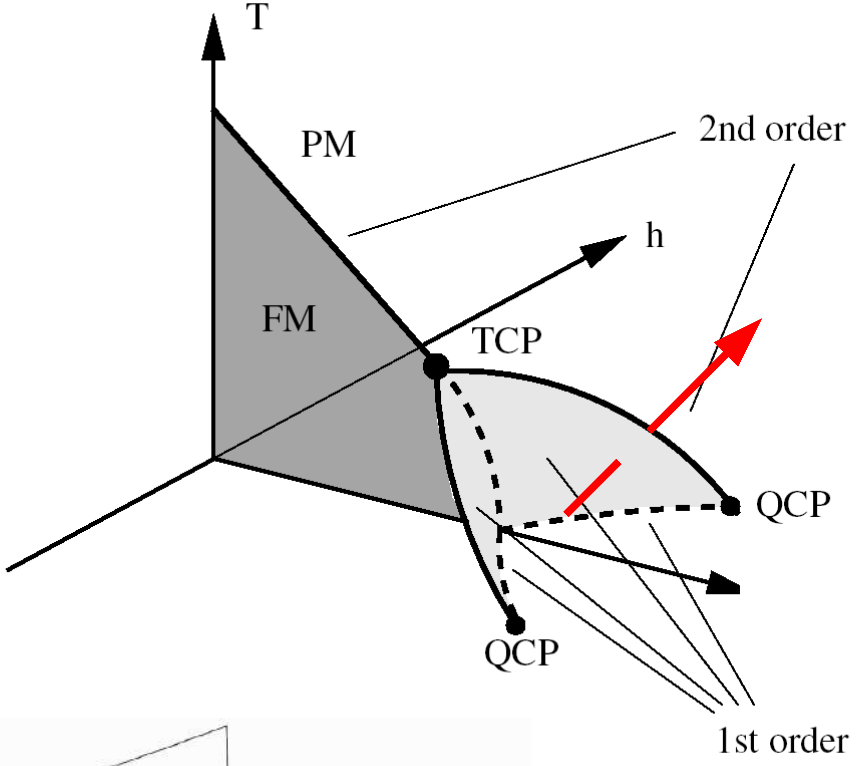
Uhlarz *et al.*, PRL 2004

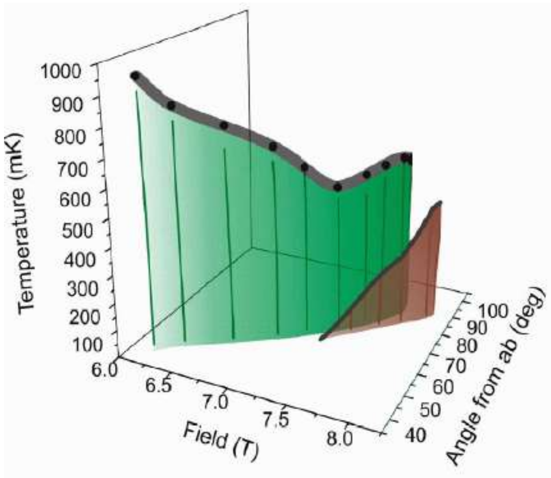
$Sr_3Ru_2O_7$

Gareth Conduit (Cavendish Laboratory)

- Resistance anomaly







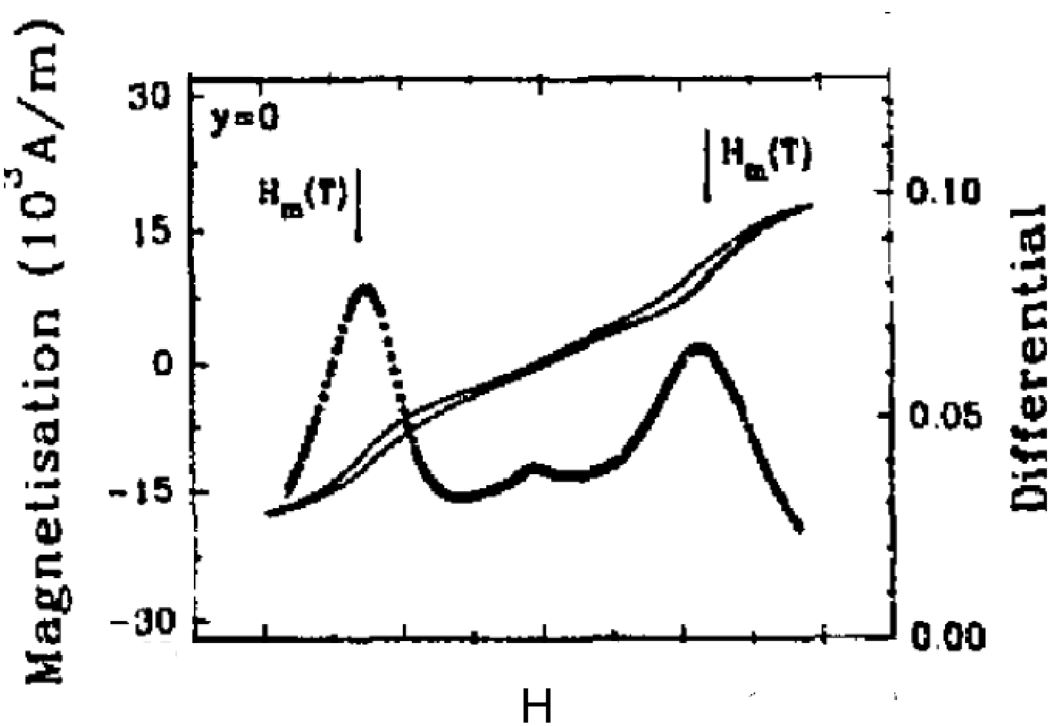
- Consistent with a new crystalline phase

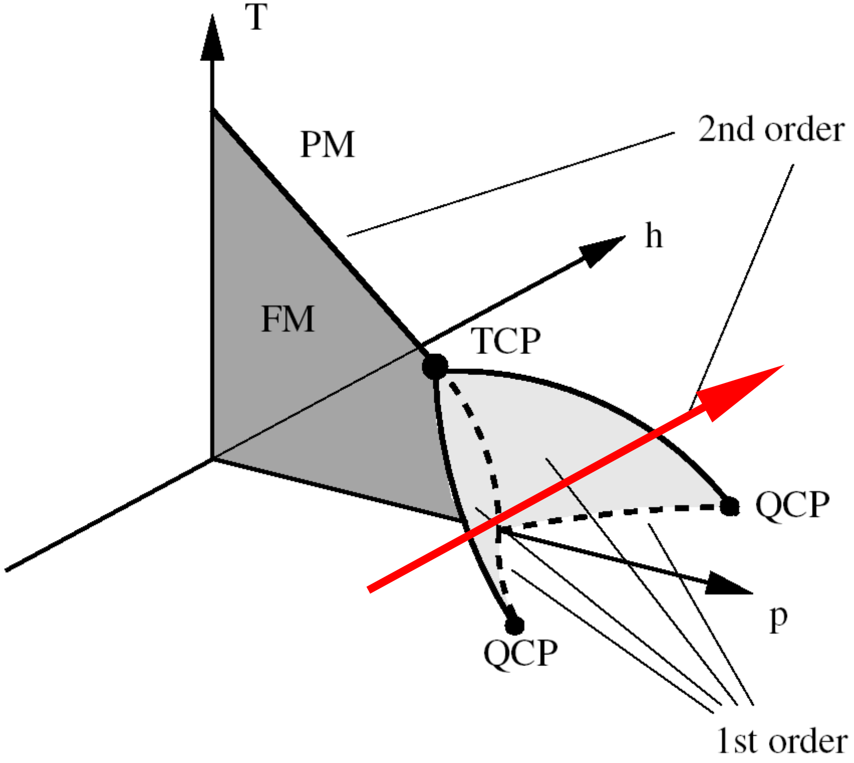
Grigera *et al.*, Science 2004

$NbFe_2$

Gareth Conduit (Cavendish Laboratory)

- $NbFe_2$ displays antiferromagnetic order where it is expected to be ferromagnetic -- could this be a textured ferromagnetic phase?



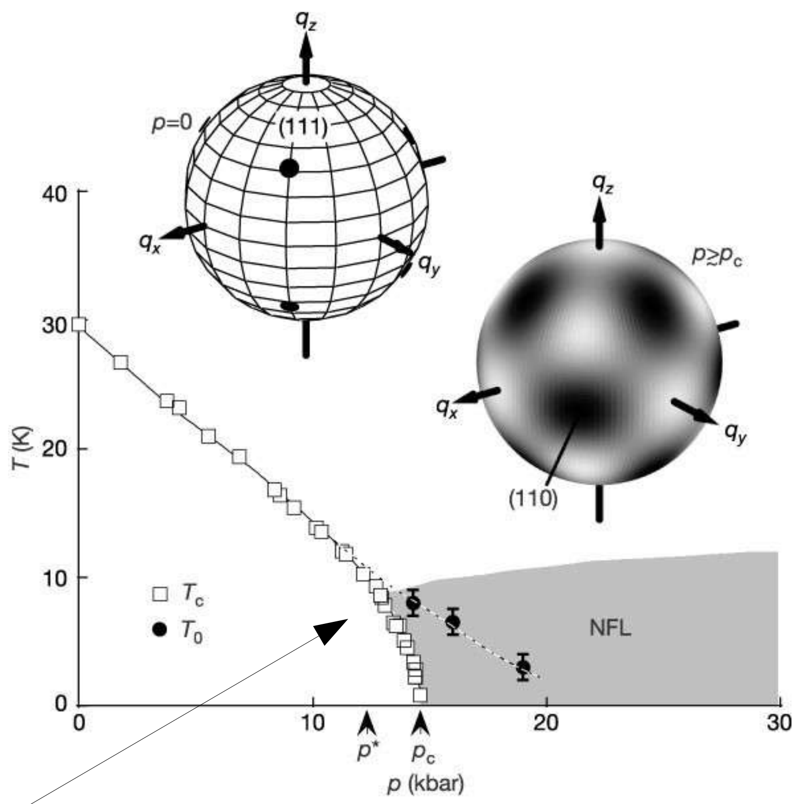


Crook & Cywinski, JMMM 1995

MnSi

Gareth Conduit (Cavendish Laboratory)

- MnSi displays non-Fermi liquid behaviour consistent with a spin state (though in a non-centrosymmetric crystal)



Tricritical point

Pfleiderer *et al.*, Nature 2004

Previous analytical work

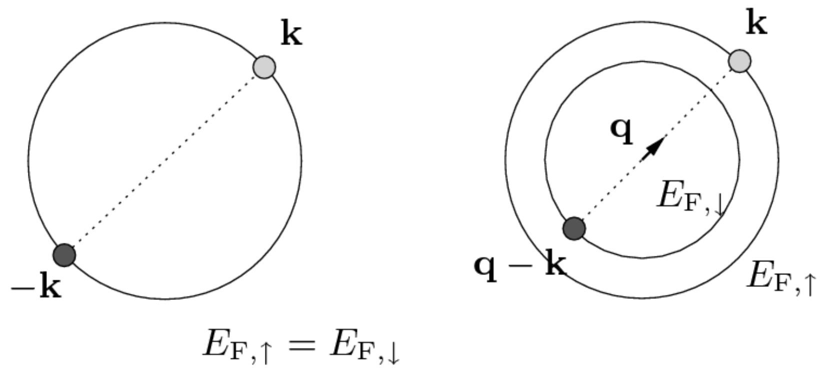
Gareth Conduit (Cavendish Laboratory)

- Current proposals exploit a quantum critical point:
- Pomeranchuk instability – Grigera *et al.*, Science 2005
- Nanoscale charge instabilities – Honerkamp, PRB 2005
- Electron nematic – Kee & Kim, PRB 2005
- Magnetic mesophase formation -- Binz *et al.*, 2005
- Here propose a spin-spiral state, previous studies focussed on non-analyticities:
- Rech, Pépin & Chubukov, PRB **74**, 195126, (2006) used Eliashberg theory
- Belitz *et al.*, PRB 1997 considered corrections due to magnetisation fluctuations

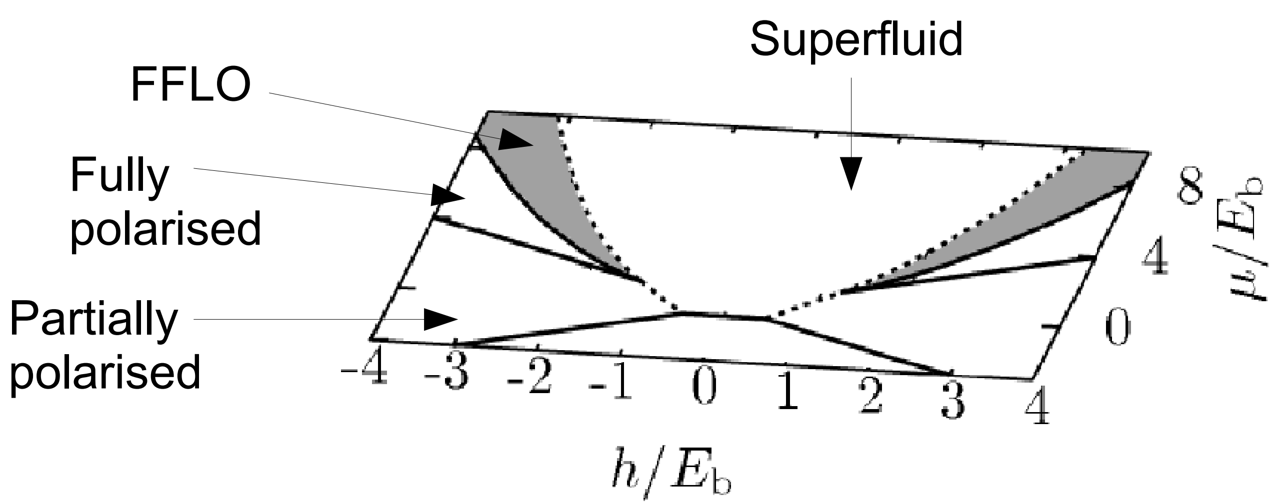
FFLO

Gareth Conduit (Cavendish Laboratory)

- The Fulde-Ferrel-Larkin-Ovchinnikov (FFLO) phase has a modulated superconducting gap



- A Cooper pair has zero momentum, with unequal Fermi surfaces the Cooper pair carries momentum, causing a modulated superconducting gap parameter Δ
- The FFLO phase preempts the normal phase-superfluid transition



GJC *et al*. PRB 2008

Ginzburg-Landau analysis

Gareth Conduit (Cavendish Laboratory)

- In analogy to FFLO[1] we can look at a Ginzburg-Landau analysis

$$\beta H = \int r m^2 + \boldsymbol{u} m^4 + v m^6 + \frac{2}{3} \boldsymbol{u} (\nabla m)^2 + \frac{3}{5} v (\nabla^2 m)^2 - hm$$

- The development of the tricritical point is accompanied by sign reversal of the gradient term
- Consider the lowest order term in a Ginzburg-Landau expansion, which is a function of the wave vector q of the textured phase

$$\beta H = \sum_q \alpha_q m_q^2$$

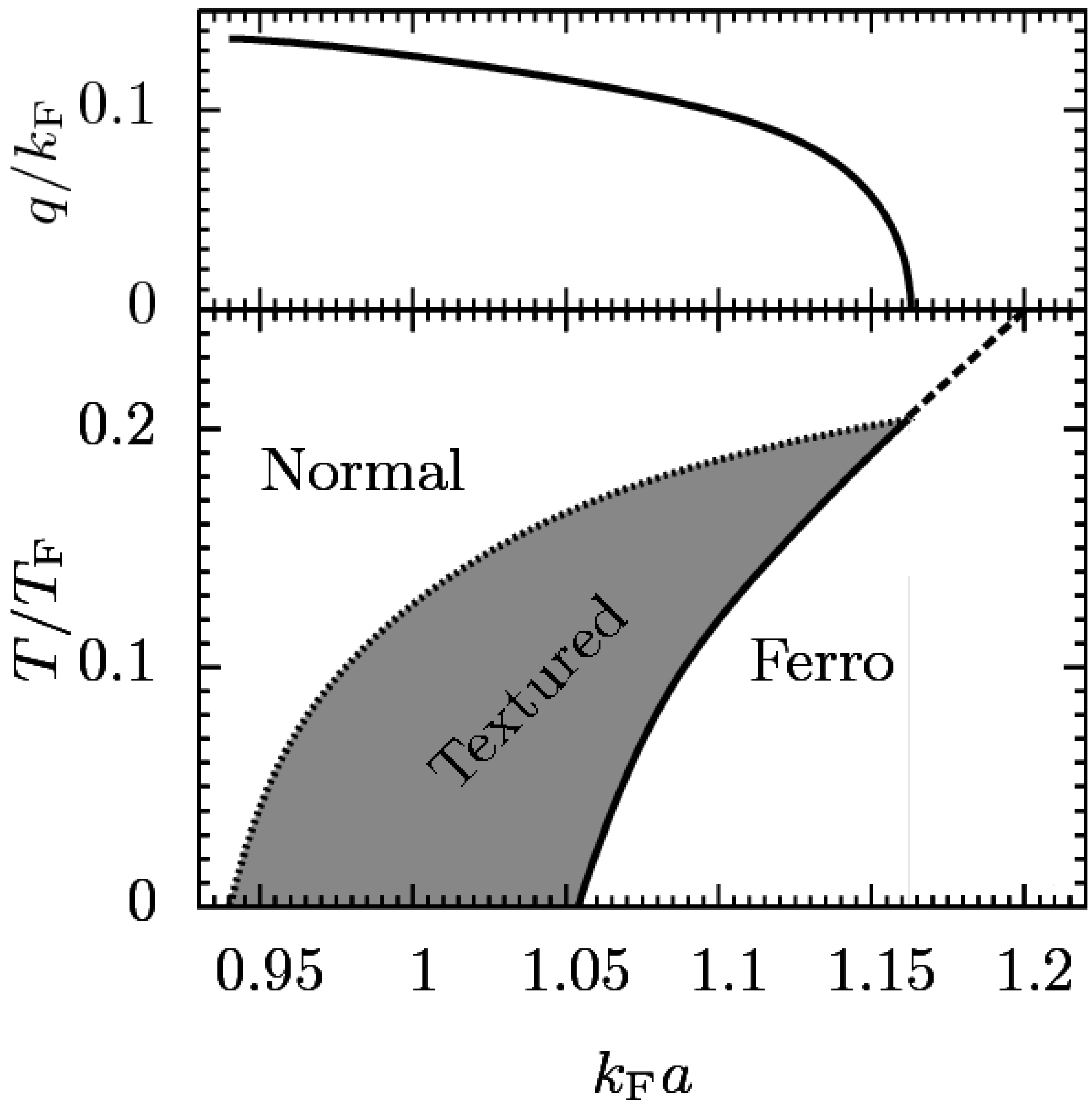
- When $\alpha_q > 0$ zero magnetisation is favourable, if $\alpha_q < 0$ a textured phase preempts the first order ferromagnetic transition

[1]Saint-James *et al*. 1969, [2]Buzdin & Kachkachi 1996

Results

Gareth Conduit (Cavendish Laboratory)

- Textured phase preempted transition with q=0.1k_F



Summary

Gareth Conduit (Cavendish Laboratory)

- Found field theoretic construction to understand population imbalance in atomic gases with a first order transition
- Confirmed with QMC calculations
- Applied improved Wohlfarth Rhodes criterion
- Ginzburg-Landau analysis of textured ferromagnetic phase
- Acknowledgements: Ben Simons & Andrew Green, EPSRC