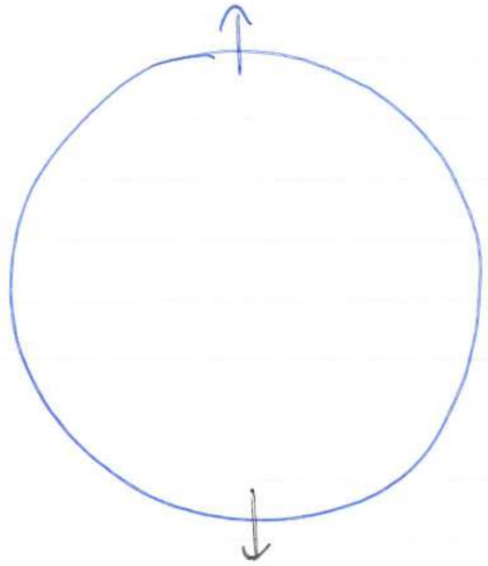
LECTURE 3

- Done repulsive interactions, now attractive interactions with $a < 0$
- Gives BCS superconductivity
- Then look at ongoing research on spin-imbalanced superconductor

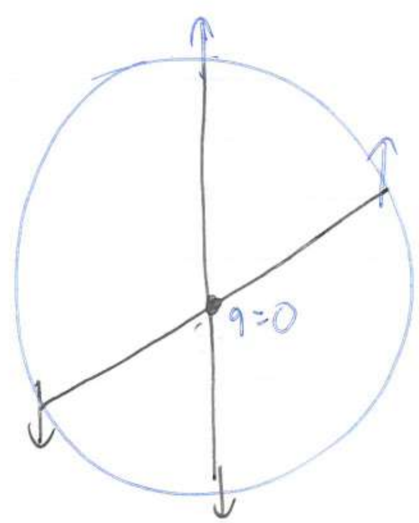
Attractive interactions : start from Fermi surface that is non-interacting



Consider two electrons, opposite spin, on top of Fermi surface

Contact interaction so only opposite spin attract.

COM momentum at $q = 0$ so construct trial wavefunction out of all orbitals sharing zero COM momentum :



KE

$$|\Psi\rangle = \sum_k \alpha_k C^+_{k\uparrow} C^+_{-k\downarrow} |FS\rangle$$

$$\langle FS | C^{+}_{p\uparrow} C^+_{-p\downarrow} | \overbrace{\hat{T}}^{\sum_{k\sigma} |k| \varepsilon'_k \, . C^+_{k\sigma} C_{k\sigma}} | \Psi \rangle$$

$$= 2.\alpha_p . \left(|p_\uparrow| + |p_\downarrow|\right) \varepsilon'_p$$

PE

$$\langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | g . \sum C^+_{k\uparrow} C^+_{-k\downarrow} C_{-k'\downarrow} C_{k'\uparrow} | \Psi \rangle$$

$$= g . \sum_k \alpha_k$$

Sum catches spherical symmetry of the Fermi surface, catching massive degeneracy

Hamiltonian

$$\langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | \hat{H} | \Psi \rangle = \langle FS | C^+_{p\uparrow} C^+_{-p\downarrow} | E | \Psi \rangle$$

$$\left(2\left(|p_\uparrow| + |p_\downarrow|\right)\varepsilon'_p - E\right) \alpha_p = g . \sum_k \alpha_k$$

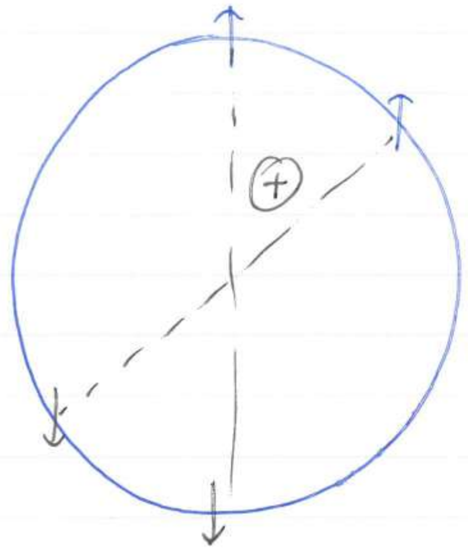
$$\sum_{p=0}^{k_0} \alpha_p = \sum_{p=0}^{k_0} \frac{g \sum_k . \alpha_k}{2(|p_\uparrow| + |p_\downarrow|)\varepsilon'_p - E}$$

$$1 = g\gamma . \sum_{p=0}^{k_0} \frac{1}{2.p.\varepsilon' - E}$$

$$E_b = 2k_0 \varepsilon' . \exp\left(-\frac{2\varepsilon'}{g\gamma}\right).$$

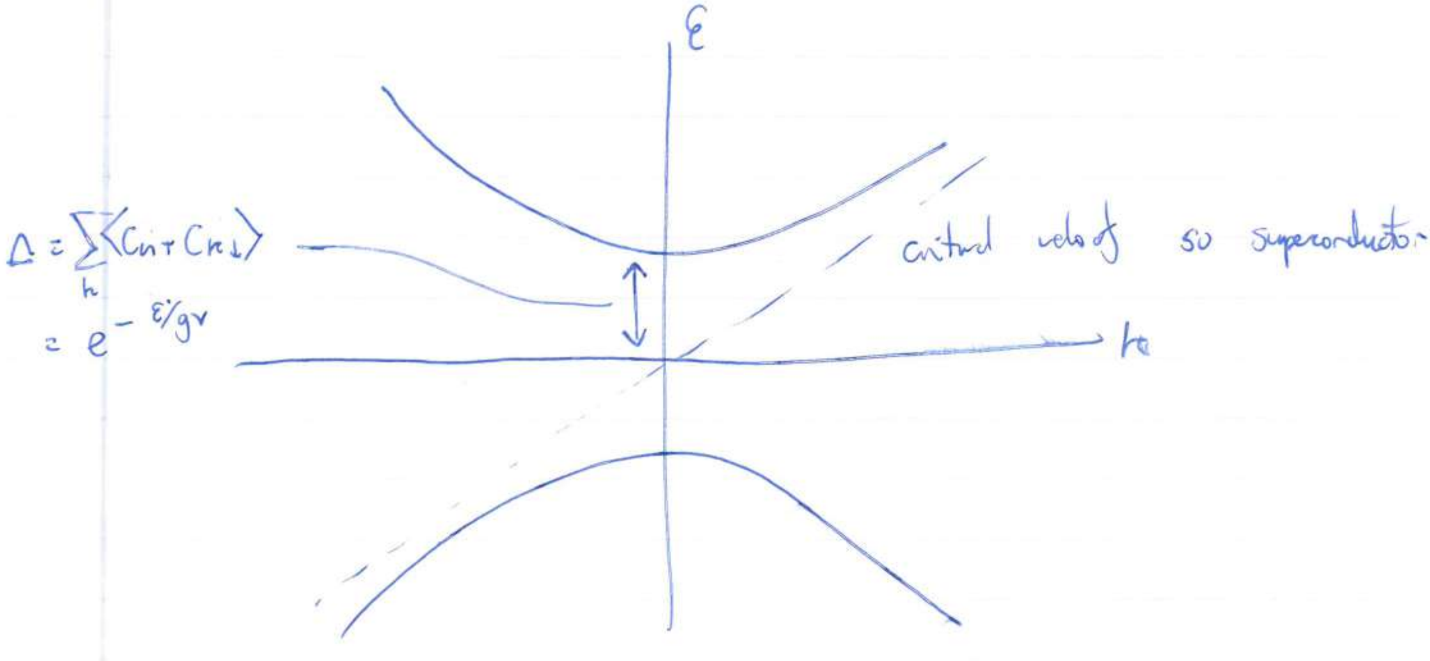
so always bound state, fundamental property of 2D quantum well (here 3D but Fermi surface removes a dimension)

Coope pair ideal building block
Once made Cooper pair can make a superconductor out of coherent state of Coope pairs



$$T_c = \hbar\omega\varepsilon' \exp\left(-\frac{\varepsilon'}{g\nu}\right)$$

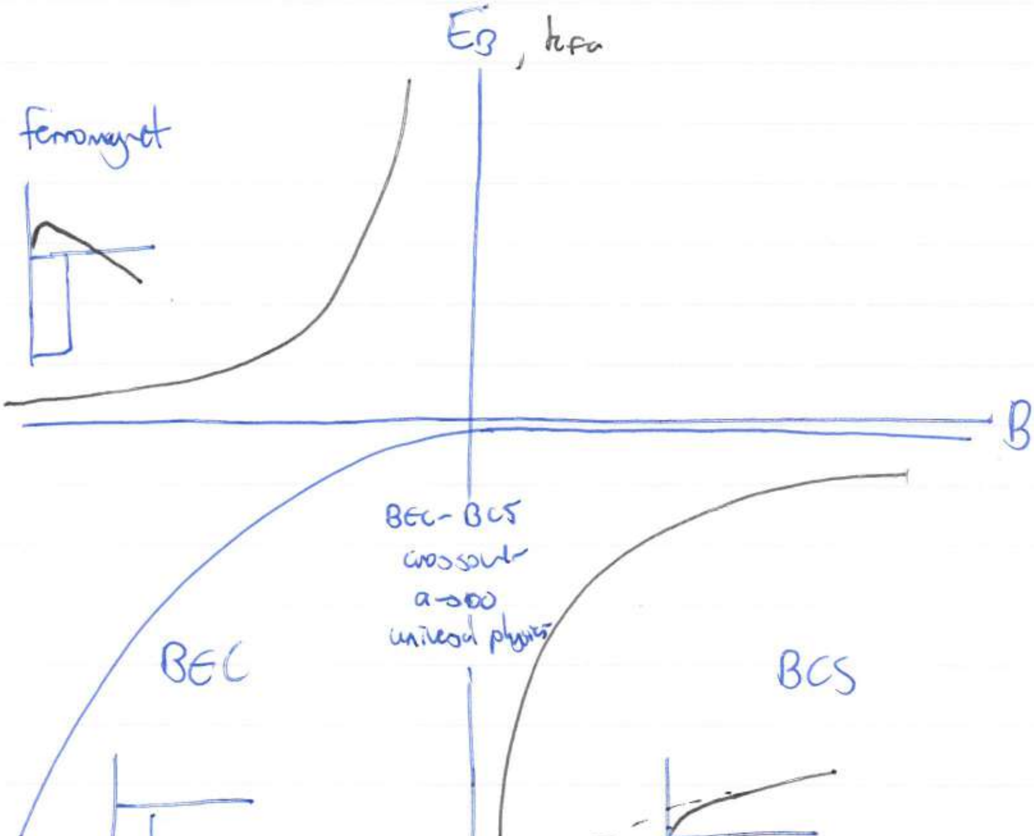
Spectrum of excitations



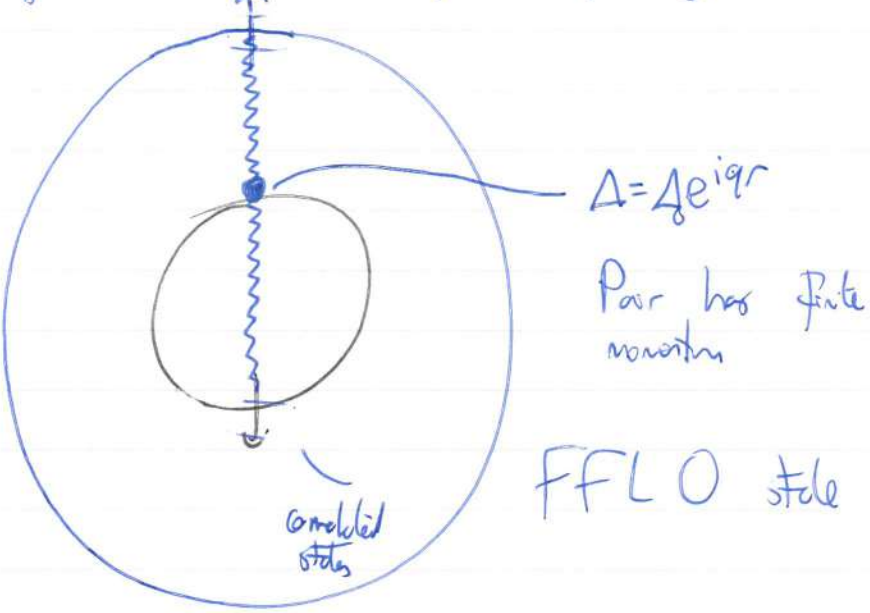
BEC - BCS crossover

BCS: coherent state of weakly bound Cooper pairs

BEC: strongly bound pairs form bosons:
(why: size much smaller than average separation of pairs)
then bosons condense into same $q=0$ state

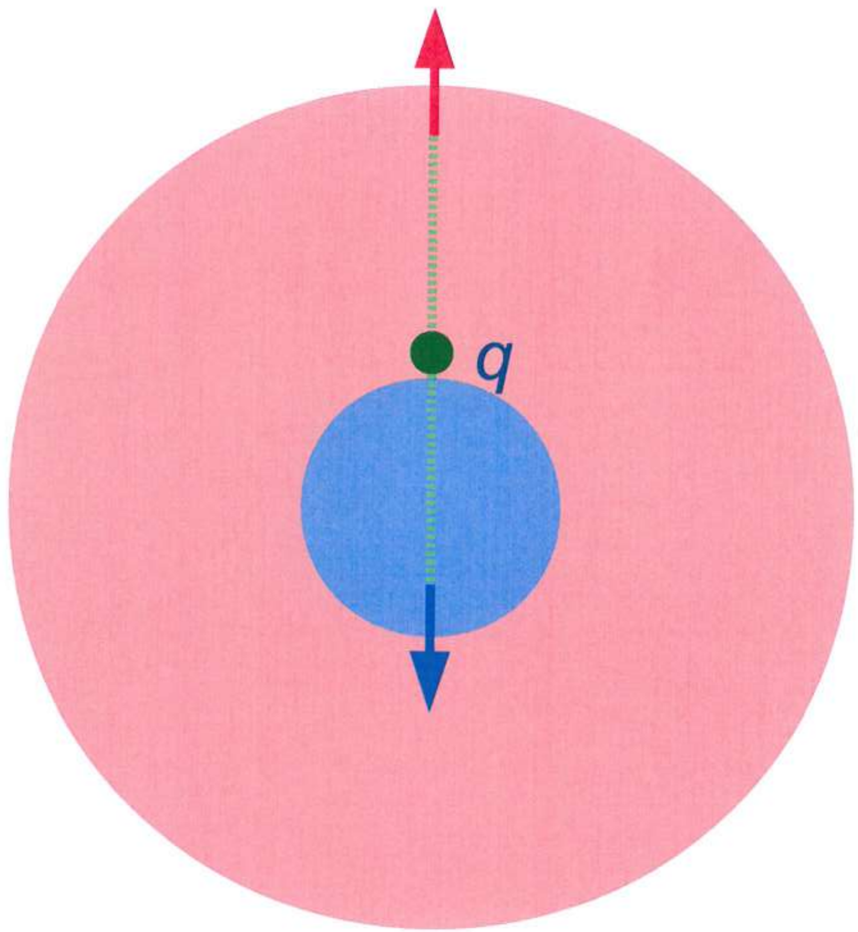


Imbalanced Fermi seas
Go through different options for pairing

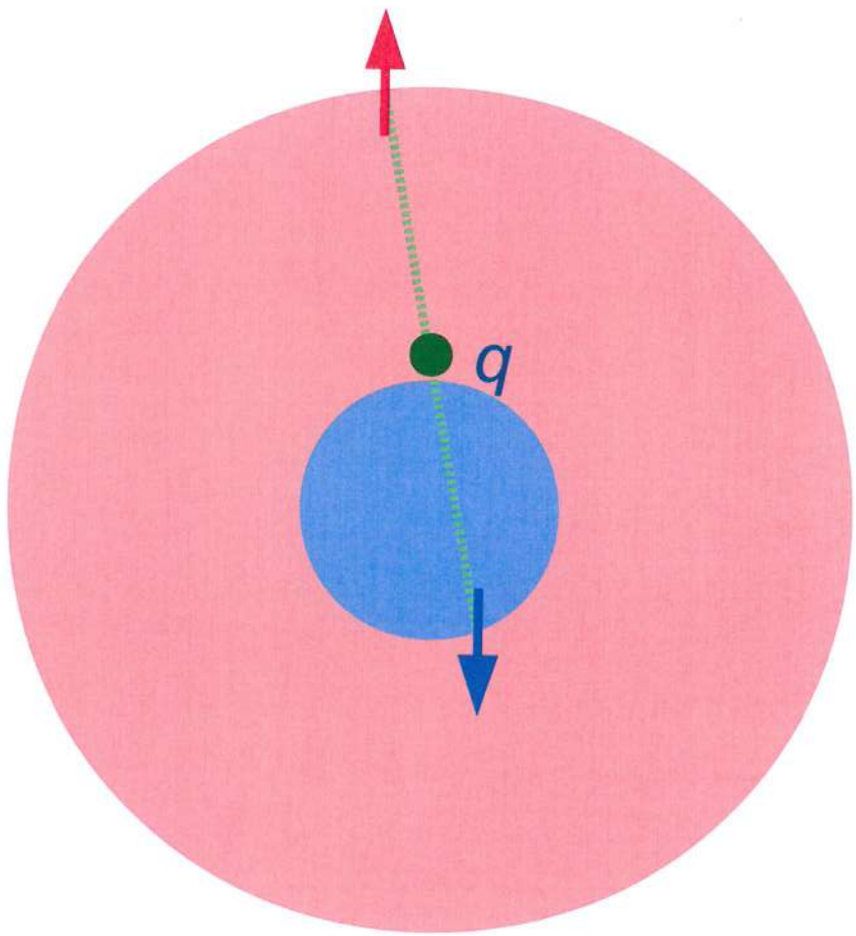


Could look for in cold atom gas
organic SC, heavy fermion materials that contain ferromagnetism and SC.

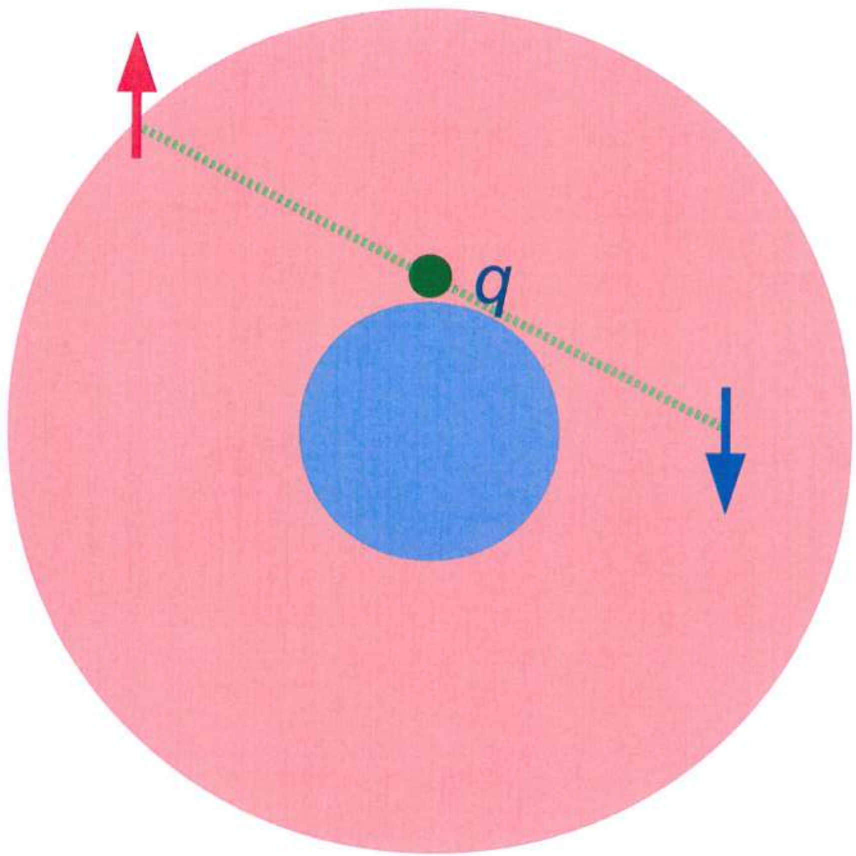
Cooper pair in a spin-imbalanced Fermi sea



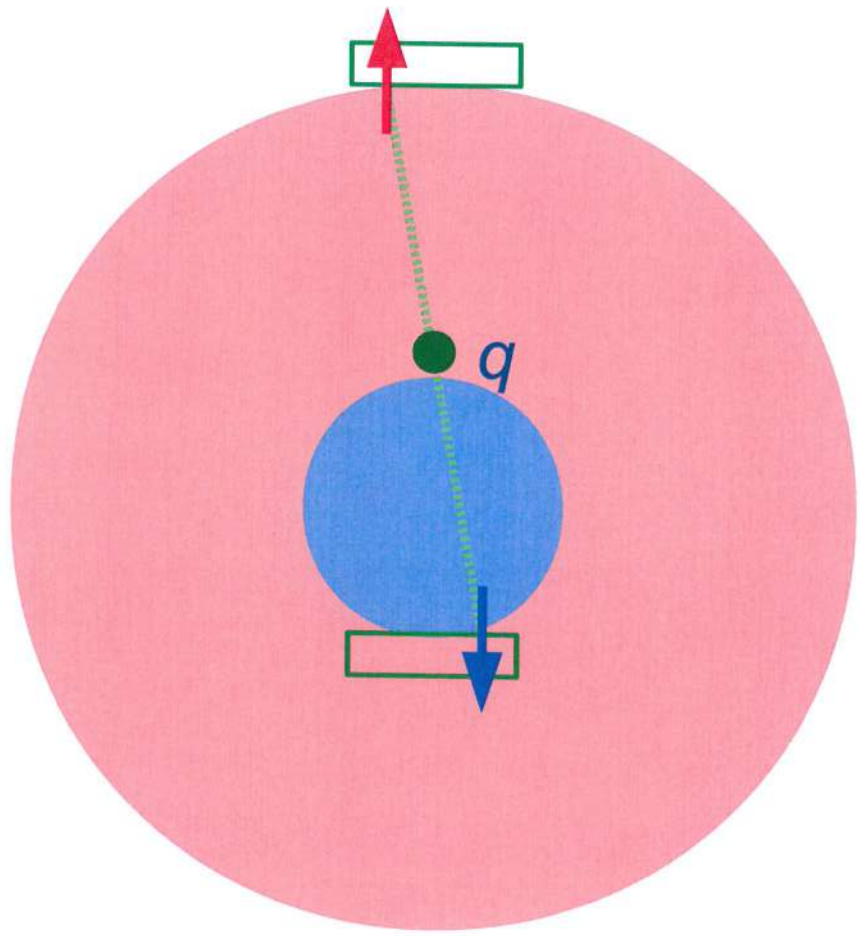
Cooper pair in a spin-imbalanced Fermi sea



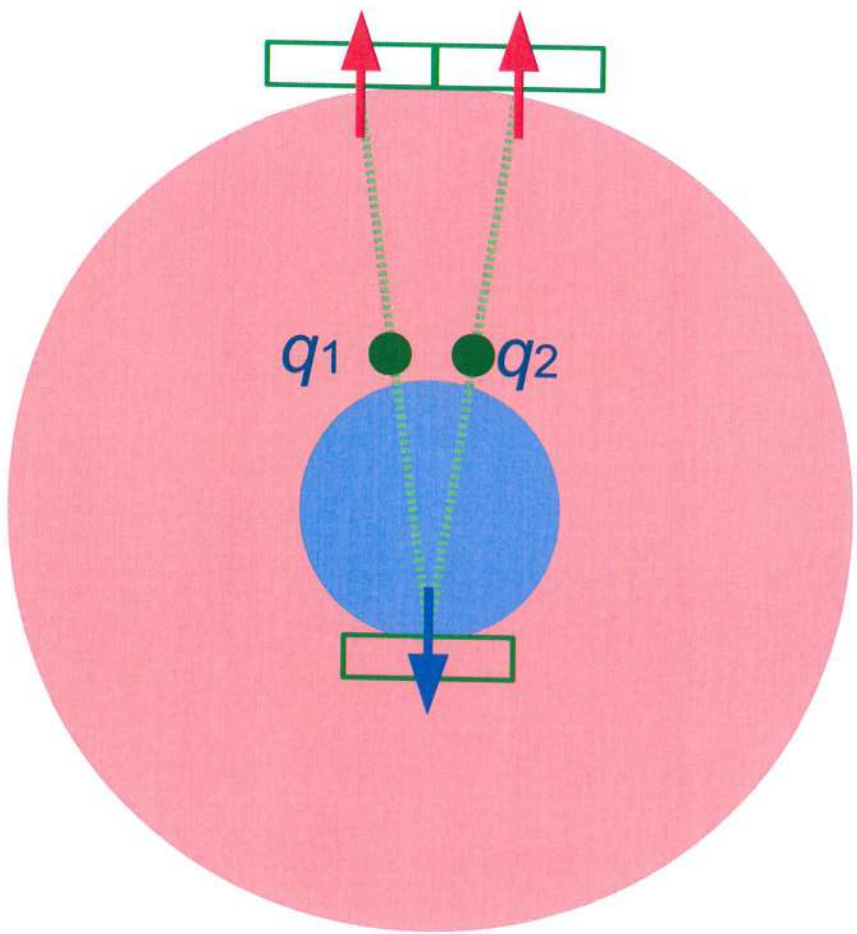
Cooper pair in a spin-imbalanced Fermi sea



States included in the wave function



Multiple majority spins in the instability



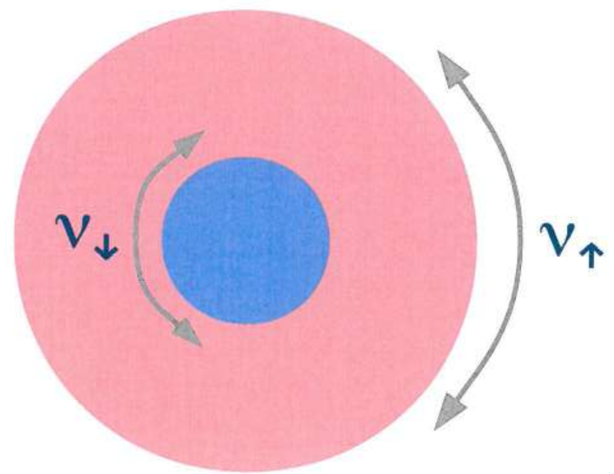
Energy of the $(N_\uparrow, N_\downarrow)$-spin instability

Binding energy of a multi-particle instability

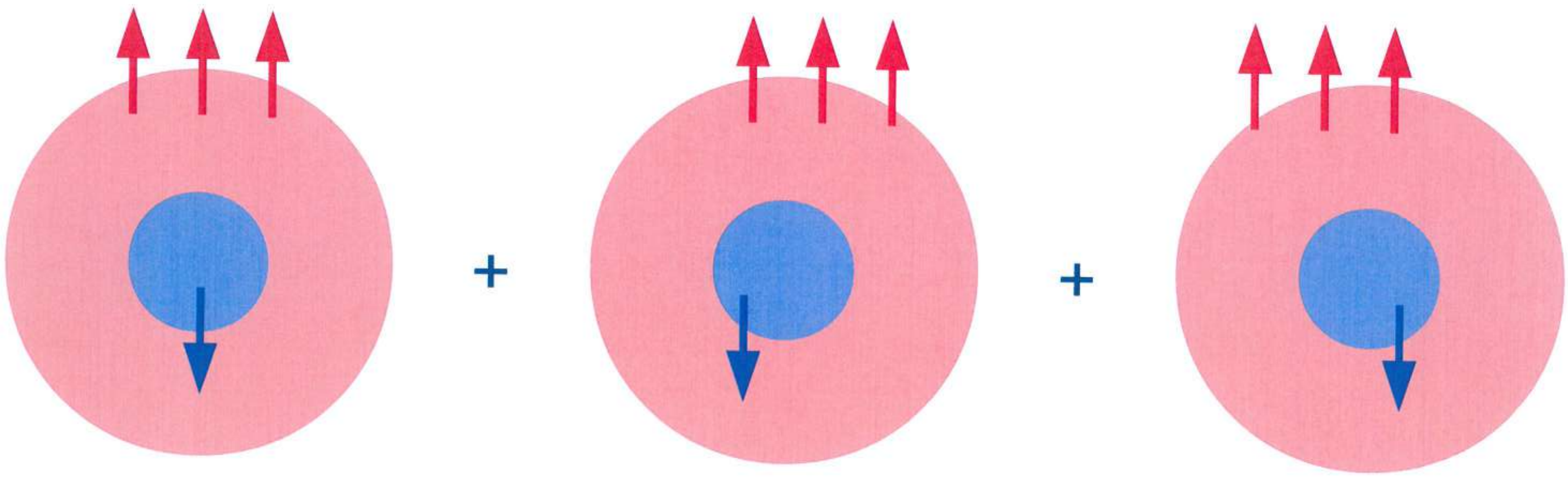
$$E = (N_\uparrow + N_\downarrow)\,\omega_D \exp\left(-\frac{(N_\uparrow + N_\downarrow)\xi'}{g N_\uparrow N_\downarrow}\frac{N_c}{\nu_c}\right) \qquad E = 2\omega_D \exp\left(-\frac{2\xi'}{g\nu}\right)$$

Optimal number of up and down spin electrons in the instability is

$$\frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow}$$



Multi-particle superconductor



Superconducting transition temperature

$$T_c = \omega_D \exp\left(-\frac{(N_\uparrow + N_\downarrow)\xi'}{2gN_\uparrow N_\downarrow} \frac{N_c}{\nu_c} \right)$$

Peak transition temperature is at the number ratio

$$\frac{N_\uparrow}{N_\downarrow} = \frac{\nu_\uparrow}{\nu_\downarrow}$$

Summary of multi-particle superconductor

Number of up to down spin electrons is the ratio of the density of states

Superconducting state based on multi-particle instability in a spin-imbalanced system

Analytical, exact diagonalization, and Diffusion Monte Carlo evidence

Applications in spin-orbit coupled systems and number fluctuations in the BCS superconductor